Sr. No. .....

ENTRANCE TEST-2023

SCHOOL OF PHYSICAL & MATHEMATICAL SCIENCE

MATHEMATICS

Question Booklet Series **A**

Total Questions : 60

Time Allowed : 70 Minutes

Roll No. : | | | | | | |

Instructions for Candidates :

1. Write your Entrance Test Roll Number in the space provided at the top of this page of Question Booklet and fill up the necessary information in the spaces provided on the OMR Answer Sheet.
2. OMR Answer Sheet has an Original Copy and a Candidate's Copy glued beneath it at the top. While making entries in the Original Copy, candidate should ensure that the two copies are aligned properly so that the entries made in the Original Copy against each item are exactly copied in the Candidate's Copy.
3. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
4. Choose the correct / most appropriate response for each question among the options A, B, C and D and darken the circle of the appropriate response completely. The incomplete darkened circle is not correctly read by the OMR Scanner and no complaint to this effect shall be entertained.
5. Use only blue/black ball point pen to darken the circle of correct/most appropriate response. In no case gel/ink pen or pencil should be used.
6. Do not darken more than one circle of options for any question. A question with more than one darkened response shall be considered wrong.
7. There will be **'Negative Marking'** for wrong answers. Each wrong answer will lead to the deduction of 0.25 marks from the total score of the candidate.
8. Only those candidates who would obtain positive score in Entrance Test Examination shall be eligible for admission.
9. Do not make any stray mark on the OMR sheet.
10. Calculators and mobiles shall not be permitted inside the examination hall.
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1. Let $A = \{x \in R : x < 1\}$ and

$B = \{x \in R : -1 < x \leq 3\}$. Then :

(A) Both A and B are bounded

(B) A is bounded above by 1 and its supremum $1 \in A$

(C) The supremum of A belongs to A and the infimum of B belongs to B

(D) The supremum of A does not belong to A and the supremum of B belongs to B

2. Which of the following is true ?

(A) The set of the real numbers is the neighbourhood of each of its points.

(B) The set Q of rational numbers is the neighbourhood of each of its points.

(C) The closed interval [a, b] is a neighbourhood of its end points a and b

(D) None of the above

3. What is the set of limit points for the set

$$S = \left\{-1, 1, -\frac{1}{2}, \frac{3}{2}, -\frac{2}{3}, \frac{4}{3}, ...\right\}$$

(A) $\left\{-\frac{1}{2}, \frac{3}{2}\right\}$

(B) $\{-1, 1\}$

(C) $\{0, -1\}$

(D) None of the above

4. Which of the following is not true ?

(A) The set of all integers if countable.

(B) The set of all ordered pairs of integers is countable.

(C) The set of all real numbers is uncountable.

(D) The set of all rational numbers is uncountable.

5. What is the limit point of the sequence $\{a_n\}$, where $a_n = \frac{(-1)^{n-1}}{n!}$, $n \in N$.

(A) $-\frac{1}{2}$

(B) -1

(C) 0

(D) None of the above

6. If $a_n = n^2$ and $b_n = -n^2$, $n \in N$, then

(A) the sequence $(a_n + b_n)$ diverges

(B) the sequence $(a_n - b_n)$ converges

(C) the sequence $\left\{\frac{a_n}{b_n}\right\}$ diverges

(D) the sequence $\left\{\frac{a_n}{b_n}\right\}$ converges

7. $\lim_{n\to\infty}\left(\frac{2n-3}{n+1}\right)$ equals

(A) 0

(B) 1

(C) 2

(D) -3

8. Which of the following is not true ?

(A) If a sequence is convergent, it converges to a unique limit.

(B) Every convergent sequence is bounded.

(C) Every bounded sequence is convergent.

(D) Every bounded monotonic sequence is convergent.

9. If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n \to 0} a_n$ equals :

(A) 0

(B) 1

(C) ∞

(D) None of the above

10. For any fixed value of n, the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ is :

(A) absolutely convergent

(B) divergent

(C) oscillates finitely

(D) oscillates infinitely

11. If $\sum a_n$ is a positive term series such that $\lim_{n \to \infty} (a_n)^{\frac{1}{n}} = l$, then the series is convergent if :

(A) $l < 1$

(B) $1 < l < 3$

(C) $l \geq 3$

(D) None of the above

12. The series $A = \sum_{n=1}^{\infty} a_n$, where $a_n = \frac{(-1)^{n+1}}{\sqrt{n}}$. Then

(A) A is convergent

(B) A is divergent

(C) $\{a_n\}$ is monotonically increasing sequence

(D) the series $|a_n|$ is convergent

13. The radius of convergence and the exact interval of convergence of the power series $\sum \frac{(n+1)x^n}{(n+2)(n+3)}$ is :

(A) R = 0, [0, 1]

(B) R = 1, [–1, 1]

(C) R = ∞

(D) None of the above

14. Let $\{f_n\}$ be a sequence of functions such that $\lim_{n \to \infty} f_n = f(x)$, $x \in [a, b]$, and let $M_n = \sup_{x \in [a, b]} |f_n(x) - f(x)|$. Then $f_n \to f$ uniformly on [a, b] if and only if :

(A) $M_n \to 0$ as $n \to \infty$

(B) $M_n \to \infty$ as $n \to \infty$

(C) $M_n \to 1$ as $n \to \infty$

(D) None of the above

15. Let $\{f_n\}$ be a sequence, where $f_n(x) = \frac{nx}{1 + n^2x^2}$, then

(A) $M_n \to 0$ as $n \to \infty$

(B) $M_n \to \frac{1}{2}$ as $n \to \infty$

(C) $M_n \to 1$ as $n \to \infty$

(D) None of the above

16. What is the identity element of the group of all integers Z with operation aob = a + b + 1, a, b $\in$ Z.

(A) 0

(B) 1

(C) –1

(D) None of the above

17. The set of all n × n non-singular matrices having their elements as rational numbers is :

(A) an infinite abelian group wrt matrix multiplication

(B) an abelian group wrt matrix addition

(C) an infinite non-abelian group wrt matrix multiplication

(D) None of the above

18. If α = (1 2 3 4 5) and β = (2 3) (4 5) be two permutations of five symbols 1, 2, 3, 4, 5 then $\alpha\beta$ equals :

(A) (1 3 5)

(B) (2 4 5)

(C) (1 2 3)

(D) None of the above

19. Which of the following is the set of generators of the cyclic group G = ({0, 1, 2, 3, 4, 5}, +6) are :

(A) {1, 2}

(B) {1, 5}

(C) {2, 5}

(D) None of the above

20. Which of the following is true ?

(A) The symmetric group P_3 of permutations of degree three is non-abelian, while its subgroup A_3 is abelian

(B) The symmetric group P_4 of permutations of degree four is abelian, while its subgroup A_4 is also abelian

(C) The symmetric group P_3 of permutations of degree three is non-abelian, while its subgroup A_3 is also non-abelian

(D) None of the above

21. Let G = {a} be a cyclic group of order six and H be the subgroup generated by a^2. Then the order of the quotient group (G/H) is :

(A) 1

(B) 2

(C) 3

(D) 4

22. Which of the following is true for the group of order 45 ?

(A) G has a normal subgroup of order 9

(B) G has an element of order 9

(C) G is a non-abelian group

(D) G has no proper subgroup of order 5

23. What is the order of the element $\frac{2}{3}+z$ in the quotient group (G/Z) of the additive group of rationals ?

(A) 2

(B) 3

(C) 5

(D) 6

24. If the quotient group (G/Z), where Z is the centre of the group G, then

(A) G is abelian

(B) G is cyclic

(C) G is non-abelian

(D) None of the above

25. The set F = {0, 1, 2, 3, 4, 5, 6} under addition and multiplication modulo 7 forms :

(A) Commutative ring with zero divisors

(B) Non-commutative ring with unity

(C) Field

(D) None of the above

26. Let R[x] be the ring of polynomials of a ring R, then which of the following statements is/are true ?

(i) R is an integral domain if and only if R[x] is an integral domain.

(ii) If R is an integral domain, then deg [f(x) g(x)] = deg f(x) + deg g(x), where f(x), g(x) $\in$ R[x].

(A) (i) only

(B) (ii) only

(C) Neither (i) nor (ii)

(D) Both (i) and (ii)

27. What is the number of proper ideals in a field F ?

(A) At least one

(B) Zero

(C) Exactly one

(D) None of the above

28. $\lim_{n\to\infty}\left(a^{\frac{1}{n}}-1\right)n$ equals :

(A) log a

(B) 1

(C) ∞

(D) None of the above

29. What is the locus of the extremity of the polar subnormal of the curve r = f(θ) ?

(A) $\frac{dr}{d\theta}$

(B) $\theta-\frac{\pi}{2}$

(C) $r=f'\left(\theta-\frac{\pi}{2}\right)$

(D) None of the above

30. What is the angle of intersection of the curves r = a(1 + cos θ), r = b(1 – cos θ) ?

(A) $\frac{\pi}{2}$

(B) 0

(C) –1

(D) π

31. What is the radius of curvature at any point of the curve $r = ae^{\theta \cot\alpha}$?

(A) r cosecα

(B) r cotα

(C) r secα

(D) None of the above

32. The asymptotes of the curve rθ = a are :

(A) r sinθ = a

(B) r cosθ = a

(C) cosθ = r^2

(D) None of the above

33. If $U = \sqrt{x^2 - y^2}\sin^{-1}\frac{y}{x}$, then $x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y}$ equals :

(A) 2U

(B) 0

(C) U

(D) None of the above

34. If $Z = x^y$, then $\frac{\partial Z}{\partial x}$ equals :

(A) $x^y \log x$

(B) $y\ x^{y-1}$

(C) $y\ e^x$

(D) None of the above

35. What is the nth-derivative of 3^{2x} ?

(A) $y_n = 3^{2x}(\log 3)^n$

(B) $y_n = 2(3^{2x})^n(\log 3)$

(C) $y_n = 2^n 3^{2x}(\log 3)^n$

(D) None of the above

36. $\frac{d}{dx}(\sin\sqrt{x})$ equals :

(A) $\cos\sqrt{x}$

(B) $\frac{1}{2\sqrt{x}}\cos\sqrt{x}$

(C) $\frac{\sqrt{x}}{2}\cos\sqrt{x}$

(D) None of the above

37. Let the functions f(x) and g(x) be defined by :

$$f(x) = \begin{cases} 1, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$$

$$g(x) = \begin{cases} 0, & \text{if } x \geq 0 \\ 1, & \text{if } x < 0 \end{cases}$$

Then

(A) f(x) is continuous at x = 0

(B) g(x) is continuous at x = 0

(C) f(x) + g(x) is continuous at x = 0

(D) f(x)g(x) is discontinuous at x = 0

38. If $x = a\cos\theta$ and $y = b\sin\theta$, then the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$ is :

(A) $-\frac{b}{a^2}$

(B) ab

(C) $-\frac{b}{a}$

(D) None of the above

39. What is the value of the function $f(x) = x^2e^x$?

(A) $4e^{-2}$

(B) 0

(C) –4

(D) None of the above

40. Rolle's theorem cannot be applied to the function f(x) = |x| in [–1, 1] because :

(A) the function is discontinuous at x = 0

(B) the function is not derivable at x = 0

(C) the function is discontinuous at $x = \frac{1}{2}$

(D) the function is not derivable at $x = -\frac{1}{2}$

41. The coefficient of x in the Maclaurin's series of $f(x) = a^x$ is :

(A) 1

(B) log a

(C) a

(D) $(\log a)^2$

42. $\int \frac{1}{x - x^3} dx$ equals :

(A) $\log\sqrt{1-x^2}$

(B) $\log x - \log\sqrt{1-x^2}$

(C) $\log x - \log(1-x^2)$

(D) $\log x + \log\sqrt{1-x^2}$

43. $\int_1^2 \log x \, dx$ equals :

(A) log 4 – 1

(B) 2 log 2

(C) 2 log 2 + 1

(D) None of the above

44. The reduction formula of $I_n = \int \tan^n x \, dx$ is

(A) $I_n = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$

(B) $I_n = \frac{\tan^{n-2} x}{n-2} - I_{n-1}$

(C) $I_n = \frac{2\tan^n x}{n} - I_{n+3}$

(D) None of the above

45. For odd n, $\int_0^{\pi} \frac{\sin nx}{\sin x} dx$ equals :

(A) 0

(B) $-\pi$

(C) π

(D) None of the above

46. What is the general solution of the differential equation

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y} \quad ?$$

(A) $y = \frac{x^3}{3} + e^x + c$

(B) $e^y = \frac{x^3}{3} + e^x + c$

(C) $y = e^x + e^{-y} + c$

(D) None of the above

47. What is the particular integral of the differential equation

$(D^2 - 3D + 2)y = e^x + e^{2x}$?

(A) $\frac{e^x}{5} + \frac{xe^{2x}}{6}$

(B) $-xe^x + \frac{xe^{2x}}{4} + c$

(C) $xe^{2x} - xe^x$

(D) None of the above

48. What is the complementary function of the differential equation

$(D^4 - D^2)y = 2$?

(A) $c_1e^x + c_2e^{-x}$

(B) $c_1 + c_2x + c_3e^x$

(C) $c_1 + c_2x + c_3e^x + c_4e^{-x}$

(D) None of the above

49. What is the general solution of the differential equation

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0 \quad ?$$

(A) $y = (c_1 + c_2 \log x)x^2$

(B) $y = (c_1e^x + c_2e^{-x})x$

(C) $y = (c_1 + c_2 \log x)$

(D) None of the above

50. What is the singular solution of the differential equation $y = px + \frac{a}{p}$, where $p = \frac{dy}{dx}$?

(A) $y^2 = 4ax$

(B) $y = 4ax^2$

(C) $y = 4ax$

(D) None of the above

51. What is the general solution of the differential equation $(px - y)^2 = p^2 + m^2$, where $p = \frac{dy}{dx}$.

(A) $y = cx \pm \sqrt{m^2 + c^2}$

(B) $y = \sqrt{cx - m}$

(C) $y = \sqrt{m^2 - 4ac}$

(D) None of the above

52. For a first order non-linear differential equation $f\left(x, y, \frac{dy}{dx}\right) = 0$, which of the following statements is/are true ?

(i) Its general solution must contain only one arbitrary constant.

(ii) Its singular solution can be obtained by substituting particular values of the arbitrary constant in its general solution.

(iii) Its singular solution is an envelope of its general solution which also satisfies the equation.

(A) (i), (ii) and (iii)

(B) (i) and (ii)

(C) (i) and (iii)

(D) (ii) and (iii)

53. Which of the following is a non-linear differential equation ?

(A) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = e^x$

(B) $\frac{dy}{dx} + 3y = e^x$

(C) $\left(\frac{dy}{dx}\right)^2 + 5y = 0$

(D) None of the above

54. Let $I = \int_{-1}^{1} P_m(x)\, P_n(x)\, dx$, where $P_m(x)$ and $P_n(x)$ are Legendre polynomials, then

(A) I = 0, if $m \neq n$

(B) I = 0, if m = n

(C) $I = \frac{2}{2n+1}$, if $m \neq n$

(D) $I = \frac{2}{2n+1}$, if $m > n$

55. For n^{th} Legendre polynomial $C_n\left[\frac{d^n(x^2-1)^n}{dx^n}\right]$, the value of C_n is :

(A) $\frac{1}{n!\,2^n}$

(B) $\frac{n!}{2^n}$

(C) $n!\ 2^n$

(D) None of the above

56. Which of the following is the Bessel's equation ?

(A) $z^2\left(\frac{d^2w}{dz^2}\right)+z\left(\frac{dw}{dz}\right)+(z^2+n^2)w=0$

(B) $z^2\left(\frac{d^2w}{dz^2}\right)-z\left(\frac{dw}{dz}\right)+(z^2-n^2)w=0$

(C) $z^2\left(\frac{d^2w}{dz^2}\right)+z\left(\frac{dw}{dz}\right)+(z^2-n^2)w=0$

(D) None of the above

57. What is the Wronskian of x and xe^2 ?

(A) 0

(B) $x - x^2e^x$

(C) x^2e^x

(D) None of the above

58. The partial differential equation obtained from $z = f(x) + e^y g(x)$ by eliminating the arbitrary functions is :

(A) p = q

(B) t = q

(C) r = s

(D) None of the above

59. What is the general solution of the partial differential equation $p - 2q = 3x^2 \sin(y + 2x)$?

(A) $x^3\sin(y + 2x) - z = \phi(y + 2x)$

(B) $\sin(y + 2x) = z$

(C) $\phi(y - 2x) = \cos(y + 2x)$

(D) None of the above

60. The partial differential equation

$$xy\frac{\partial^2u}{\partial x^2}-(x^2-y^2)\left(\frac{\partial^2u}{\partial x\partial y}\right)-xy\left(\frac{\partial^2u}{\partial y^2}\right)+y\frac{\partial u}{\partial x}-x\frac{\partial u}{\partial y}=2(x^2-y^2)$$

(A) is parabolic at all points

(B) is hyperbolic at all points

(C) is elliptic at all points

(D) None of the above

ROUGH WORK

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Sr. No. 453

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SCHOOL OF PHYSICAL & MATHEMATICAL SCIENCES

MATHEMATICS

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Question Booklet Series A

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SEAL

1. $\lim_{x\to 0}\frac{e^{\sin x}-1}{x}$ equals :

(A) 0

(B) –1

(C) e

(D) 1

2. Let $f(x)=\begin{cases}1, & x\geq 0\\ -1, & x<0.\end{cases}$

$g(x)=\begin{cases}-1, & x\geq 0\\ 1, & x<0.\end{cases}$

Then which of the following is true at x = 0 ?

(A) f and g are continuous

(B) fg is discontinuous

(C) f + g is continuous

(D) None of the above

3. If $z=x^4y^2\sin^{-1}\left(\frac{y}{x}\right)$, then $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}$ equals :

(A) 6z

(B) 3z

(C) 9z

(D) 6

4. Let $z = x^2 - 3y^2 + 20$, where $x = 2\cos t$ and $y = 2\sin t$. At $t=\frac{\pi}{4}$, $\frac{dz}{dt}$ equals :

(A) 16

(B) –16

(C) –8

(D) 8

5. What is the radius of curvature of the curve $s = a\sec\psi\tan\psi + a\log(\sec\psi + \tan\psi)$ at the point (s, ψ) ?

(A) $2a\sec^3\psi$

(B) $a\sec^3\psi$

(C) $2\sec^3\psi$

(D) None of the above

6. What is the angle between the radius vector and the tangent to the curve $r = a(1 + \sin\theta)$ at $\theta = \frac{\pi}{6}$?

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) None of the above

7. What is the pedal equation of the curve $a^2 = r^2\cos 2\theta$?

(A) $a^2 = rp$

(B) $a = rp$

(C) $a = r + p$

(D) None of the above

8. What is the number of asymptotes of the curve $r=\frac{a}{1-\cos\theta}$?

(A) 2

(B) 1

(C) 0

(D) None of the above

9. On which of the following functions Rolle's theorem cannot be applied ?

(A) $f(x) = \sin x$ in $[0, \pi]$

(B) $f(x) = \sqrt{4-x^2}$ in $[-2, 2]$

(C) $f(x) = |x|$ in $[-1, 1]$

(D) $f(x) = x^2$ in $[-1, 1]$

10. In which of the following intervals, all the conditions of the Lagrange's mean value theorem is satisfied ?

(A) [2, 4]

(B) [1, 3]

(C) [–2, 2]

(D) None of the above

11. $\lim\limits_{x\to 0}\left(\dfrac{\ln(1+x)}{\sin x}\right)$ equals :

(A) –1

(B) 1

(C) 0

(D) None of the above

12. What is the maximum value of the function $f(x) = 2x^3 - 9x^2 - 24x - 20$?

(A) 1

(B) 7

(C) –7

(D) None of the above

13. $\int \dfrac{dx}{\sqrt{2x-x^2}}$ equals :

(A) $\cos^{-1}(x - 1)$

(B) $\sin^{-1}(x - 1)$

(C) $\sin(x - 1)$

(D) $\cos(x - 1)$

14. $\int \dfrac{e^x}{(e^x-1)(e^x+2)}dx$ equals :

(A) $\ln\left|\dfrac{e^x+1}{e^x+2}\right|^{\frac{1}{3}}$

(B) $\ln\left|\dfrac{e^x-1}{e^x+2}\right|^{\frac{1}{3}}$

(C) $\ln\left|\dfrac{e^x-1}{e^x-2}\right|^{\frac{1}{3}}$

(D) None of the above

15. $\dfrac{d}{dx}\int_0^{\cos x}(t^4+6)\,dt$ equals :

(A) $-x(\cos^4 x + 6)\sin x$

(B) $-x(\cos^4 x + 6)\cos x$

(C) $-(\cos^4 x + 6)\cos x$

(D) None of the above

16. The differential equation $(ay^2 + x + x^8)dx + (y^8 - y + bxy)dy = 0$ is exact if :

(A) b = a

(B) a = 1 and b = 3

(C) a = 3 and b = 1

(D) b = 2a

17. What is the integrating factor for the differential equation

$$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2 \ ?$$

(A) $\frac{1}{1+x^2}$

(B) $\frac{1}{1+x}$

(C) $\frac{1}{1-x}$

(D) $1 + x$

18. What is the particular integral for the differential equation

$$(D + 2)(D - 1)^3y = (e^x + 1) \ ?$$

(A) $\frac{1}{18}x^3e^x - \frac{1}{2}$

(B) $18x^3e^x - 2$

(C) $\frac{1}{18}x^2e^x - \frac{1}{2}$

(D) None of the above

19. What is the general solution of the differential equation

$$(D^2 + 1)y = \cos 2x + e^{-x} \ ?$$

(A) $c_1\cos x + c_2\sin x - \frac{1}{3}\cos 3x + \frac{1}{2}e^{-x}$

(B) $(c_1 + c_2x)e^x + \frac{1}{3}\cos x + \frac{1}{2}e^{-x}$

(C) $c_1\cos x + c_2\sin x - \frac{1}{3}\cos 2x + \frac{1}{2}e^{-x}$

(D) None of the above

20. What is the general solution of the different equation

$$\left(\frac{dy}{dx}\right)^2 - 7\left(\frac{dy}{dx}\right) + 12 = 0 \ ?$$

(A) $(y - 3x - c)(y - 4x - c) = 0$

(B) $(y + 3x - c)(y - 4x - c) = 0$

(C) $(y - 3x - c)(y + 4x - c) = 0$

(D) None of the above

21. What is the general solution of the differen equation $p = \log(px - y)$, where $p = \frac{dy}{dx}$?

(A) $y = cx - e^x$

(B) $y = cx - e^c$

(C) $y = c - e^x$

(D) None of the above

22. If α is the degree and β is the order of differential equation

$$\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}} = \left(y + \frac{dy}{dx}\right)^{\frac{1}{2}}.$$

Then $3\alpha + 5\beta$ equals :

(A) 29

(B) 18

(C) 28

(D) None of the above

23. The differential equation $(y - px)^2 = a^2(1+$ where $p = \frac{dy}{dx}$, is :

(A) Linear equation

(B) Lagrange's equation

(C) Clairaut's equation

(D) None of the above

24. What is the expansion of x^2 in terms of the Legendre polynomial ?

(A) $\frac{1}{2}[2p_2(x)+p_0(x)]$

(B) $\frac{1}{3}[2p_2(x)+3p_0(x)]$

(C) $\frac{1}{3}[2p_2(x)+p_0(x)]$

(D) None of the above

25. If $J_n(x)$ is the Bessel function, then $J_{-\frac{1}{2}}(x)$ equals :

(A) $\frac{2}{\pi x}$

(B) $\frac{\pi x}{2}$

(C) $\sqrt{\frac{2}{\pi x}}\sin x$

(D) None of the above

26. What is the general solution of the Bessel's differential equation

$$x^2\left(\frac{d^2y}{dx^2}\right)+x\left(\frac{dy}{dx}\right)+\left(x^2-\frac{9}{16}\right)y=0.$$

(A) $y=AJ_{\frac{9}{16}}(x)+BJ_{-\frac{9}{16}}(x)$

(B) $y=AJ_{\frac{3}{4}}(x)+BJ_{-\frac{3}{4}}(x)$

(C) $y=AJ_{\frac{4}{3}}(x)+BJ_{-\frac{4}{3}}(x)$

(D) None of the above

27. What is the partial differential equation obtained from $z = f(x^2 - y^2)$ by eliminating the arbitrary function ?

(A) $y\frac{\partial z}{\partial x}=x\frac{\partial z}{\partial y}$

(B) $x\frac{\partial z}{\partial x}=y\frac{\partial z}{\partial y}$

(C) $y\frac{\partial z}{\partial x}=z\frac{\partial z}{\partial y}$

(D) None of the above

28. What is the general solution of the partial differential equation $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=\sin x$?

(A) $\phi(x - y, z + \cos x) = 0$

(B) $\phi(xy, z + \cos x) = 0$

(C) $\phi(x - y, z + \sin x) = 0$

(D) $\phi(x - y, z\cos x) = 0$

29. What is the complete integral of the partial differential equation $z = px + qy - 2\sqrt{pq}$.

(A) $z = (ax + by)^2 - 2\sqrt{ab}$

(B) $z = axy - 2\sqrt{ab}$

(C) $z = ax + by - 2\sqrt{ab}$

(D) None of the above

30. At all points, the partial differential equation

$$\frac{\partial^2 u}{\partial x^2}+4\left(\frac{\partial^2 u}{\partial x\partial y}\right)+4\left(\frac{\partial^2 u}{\partial y^2}\right)=0$$

(A) is hyperbolic

(B) is elliptic

(C) is parabolic

(D) None of the above

31. Which of the following statements is/are true ?

(1) The set $\left\{-1, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, \ldots\right\}$ is bounded.

(2) The set $\left\{\frac{1}{3}, \frac{4}{5}, \frac{9}{7}, \frac{16}{9}, \ldots\right\}$ is bounded.

(A) Both (1) and (2) are true

(B) Only (1) is true

(C) Only (2) is true

(D) Neither (1) is true nor (2) is true

32. Which of the following is order complete ?

(A) set of rational numbers

(B) set of irrational numbers

(C) set of integers

(D) set of real numbers

33. What is number of limit points in a finite set ?

(A) 1

(B) 2

(C) infinite

(D) None of the above

34. Which of the following is uncountable ?

(A) set of all integers

(B) set P_n of all polynomial functions with integer coefficients

(C) set {1, 4, 9, 16, ...}

(D) set of all real numbers

35. What is the number of limit points of the seque {1, 2, 1, 4, 1, 6, 1, ...} ?

(A) one

(B) more than one but finite

(C) infinite

(D) 0

36. The sequence $\left\{1+\frac{1}{n}\right\}$

(A) converges to zero

(B) converges to 1

(C) converges to 2

(D) diverges to infinity

37. If $\lim_{n\to\infty} a_n = 9$, then $\lim_{n\to\infty}\left(\frac{(a_1 + a_2 + \ldots + a_n}{n}\right.$

equals :

(A) 0

(B) 3

(C) 9

(D) None of the above

38. Which of the following is NOT true ?

(A) A necessary and sufficient condition the convergence of a monotonic seque is that it is bounded.

(B) Every monotonic decreasing sequence wh is bounded below diverges.

(C) Every monotonic increasing sequence wh is not bounded above diverges.

(D) A monotonic increasing bounded ab sequence converges.

39. The series $\frac{1.2}{3^2.4^2}+\frac{3.4}{5^2.6^2}+\frac{5.6}{7^2.8^2}+.....$

(A) oscillates finitely

(B) oscillates infinitely

(C) is divergent

(D) is convergent

40. Which of the following statements is true ?

(1) the infinite series $\sum\frac{1}{n^{1+\frac{1}{n}}}$ is divergent

(2) the infinite series $\sum\frac{1}{n^{2+\frac{1}{n}}}$ is convergent

(A) Both (1) and (2) are true

(B) Only (1) is true

(C) Only (2) is true

(D) Neither (1) is true nor (2) is true

41. Which of the following statements is true ?

(1) The infinite series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+....$ is conditionally convergent

(2) The infinite series $1-\frac{1}{2^3}+\frac{1}{3^3}-\frac{1}{4^3}+....$ is absolutely convergent

(A) Both (1) and (2) are true

(B) Only (1) is true

(C) Only (2) is true

(D) Neither (1) is true nor (2) is true

42. If $\sum U_n$ is a positive term series such that $\lim_{n\to\infty}(U_n)^{\frac{1}{n}}=t$, then the series converges if

(A) $t = 1$

(B) $t > 1$

(C) $t < 1$

(D) $t = 2$

43. Which of the following statements is/are true for the sequence $F_n(x) = x^n$ on $[0, 1]$?

(1) Uniformly convergent

(2) Pointwise convergent

(A) Both (1) and (2)

(B) Only (1)

(C) Only (2)

(D) Neither (1) nor (2)

44. If a sequence $\{f_n\}$ converges uniformly in $[a, b]$ and x_0 is a point of $[a, b]$ such that $\lim_{x\to x_0} f_n(x) = a_n$, $n = 1, 2, 3, ..., then :

(A) $\{a_n\}$ diverges

(B) $\{a_n\}$ converges

(C) $\lim_{x\to x_0} f(x) \neq \lim_{n\to\infty} a_n$

(D) $\lim_{x\to x_0} f(x) \neq a_n$

45. If R is the radius of convergence and β is the exact interval of convergence of the power series $\sum\frac{(x-1)^n}{2^n}$, then :

(A) $R = 2$ and $\beta = (-1, 3)$

(B) $R = 4$ and $\beta = (-1, 5)$

(C) $R = \infty$ and $\beta = (-1, 3)$

(D) None of the above

46. Which of the following forms a group ?

(A) {1, 2, 3} under multiplication modulo 4

(B) {1, –1, i, –i} under addition

(C) {0, 1, 2, 3} under multiplication modulo 4

(D) {1, 2, 3, 4} multiplication modulo 5

47. In the group $GL(2, Z_{11})$, the inverse of $\begin{pmatrix} 2 & 6 \\ 5 & 5 \end{pmatrix}$ is $\begin{pmatrix} k & k \\ k+1 & k-1 \end{pmatrix}$, then $5k + 3$ equals :

(A) 45

(B) 46

(C) 47

(D) None of the above

48. In the multiplicative group {1, –1, i, –i}, if x is the inverse of –1 and y is the inverse of i, then xy equals :

(A) i

(B) –i

(C) 1

(D) –1

49. For an abelian group G with a, b ∈ G and n a non-negative integer, which of the following statements is/are true ?

(1) $(ab)^n = a^n b^n$

(2) $(ab)^{-1} = a^{-1}b^{-1}$

(A) Both (1) and (2)

(B) Only (1)

(C) Only (2)

(D) Neither (1) nor (2)

50. Which of the following is NOT true for a group G of order 7 ?

(A) G is cyclic

(B) G is abelian

(C) G has no proper subgroups

(D) G has a unique proper subgroup

51. What is the number of proper subgroups of a finite cyclic group of order 12 ?

(A) One

(B) Two

(C) Four

(D) Six

52. What is the order of element 3 in the group $(\{0, 1, 2, 3, 4\}, +5)$?

(A) 4

(B) 5

(C) 6

(D) None of the above

53. Which of the following statements is/are true ?

(1) The intersection of any collection of normal subgroups is itself a normal subgroup.

(2) If H and K are subgroups of an abelian group G, then HK is a subgroup of G.

(A) Both (1) and (2)

(B) Only (1)

(C) Only (2)

(D) Neither (1) nor (2)

54. Which of the following statements is/are true ?

(1) A subgroup H of index 2 in a group G is a normal subgroup of G.

(2) If H and N are subgroups of a group G, with N normal in G, then $H \cap N$ is normal in G.

(A) Only (1)

(B) Only (2)

(C) Both (1) and (2)

(D) Neither (1) nor (2)

55. Which one of the following is NOT true ?

(A) A quotient group of an abelian group is abelian.

(B) A quotient group of a cyclic group is cyclic.

(C) The quotient group P_3/A_3 is an abelian group, where P_3 is the symmetric group of degree 3 and A_3 is the alternating group of order 3.

(D) The symmetric group P_3 of degree 3 is abelian.

56. The set M of all nxn matrices with their elements as real numbers is a :

(A) Commutative ring with unity

(B) Non-commutative ring with unity

(C) Non-commutative ring without unity

(D) Ring without zero divisors

57. The ring of integers is :

(A) A field

(B) An integral domain

(C) A division ring

(D) None of the above

58. The ring of all 2×2 matrices over rationals is :

(A) An integral domain

(B) A commutative ring

(C) A field

(D) None of the above

59. Which of the following is NOT order of a finite field ?

(A) 81

(B) 125

(C) 32

(D) 36

60. Which of the following statements is/are true ?

(1) A ring has more than one unity.

(2) A unit of a ring divides every element of the ring.

(A) Both (1) and (2)

(B) Only (1)

(C) Only (2)

(D) Neither (1) nor (2)

Sr. No. .....

ENTRANCE TEST-2021

SCHOOL OF PHYSICAL & MATHEMATICAL SCIENCES

MATHEMATICS

Question Booklet Series **A**

Total Questions : 60

Time Allowed : 70 Minutes

Roll No. : | | | | | | |

Instructions for Candidates :

1. Write your Entrance Test Roll Number in the space provided at the top of this page of Question Booklet and fill up the necessary information in the spaces provided on the OMR Answer Sheet.
2. OMR Answer Sheet has an Original Copy and a Candidate's Copy glued beneath it at the top. While making entries in the Original Copy, candidate should ensure that the two copies are aligned properly so that the entries made in the Original Copy against each item are exactly copied in the Candidate's Copy.
3. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
4. Choose the correct / most appropriate response for each question among the options A, B, C and D and darken the circle of the appropriate response completely. The incomplete darkened circle is not correctly read by the OMR Scanner and no complaint to this effect shall be entertained.
5. Use only blue/black ball point pen to darken the circle of correct/most appropriate response. In no case gel/ink pen or pencil should be used.
6. Do not darken more than one circle of options for any question. A question with more than one darkened response shall be considered wrong.
7. There will be **'Negative Marking'** for wrong answers. Each wrong answer will lead to the deduction of 0.25 marks from the total score of the candidate.
8. Only those candidates who would obtain positive score in Entrance Test Examination shall be eligible for admission.
9. Do not make any stray mark on the OMR sheet.
10. Calculators and mobiles shall not be permitted inside the examination hall.
11. Rough work, if any, should be done on the blank sheets provided with the question booklet.
12. OMR Answer Sheet must be handled carefully and it should not be folded or mutilated in which case it will not be evaluated.
13. Ensure that your OMR Answer Sheet has been signed by the Invigilator and the candidate himself/ herself.
14. At the end of the examination, hand over the OMR Answer Sheet to the invigilator who will first tear off the original OMR sheet in presence of the Candidate and hand over the Candidate's Copy to the candidate.

1. The infimum and supremum of the set $\left\{1+\frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$ is :

(A) –1, 1

(B) 0, 1

(C) –1, $\frac{2}{3}$

(D) 0, $\frac{2}{3}$

2. For any positive real number *a* there exists a positive integer *n* such that :

(A) $n > a$

(B) $n < a$

(C) $n = a$

(D) None of the above

3. If A = {1, 2, 3 ... n} then the set of limit points of A is :

(A) Countable

(B) Uncountable

(C) Empty

(D) Singleton

4. A subset S of real numbers has a limit point if :

(A) S is finite

(B) S is countably infinite

(C) S is infinite and bounded

(D) S is uncountable

5. The sequence (ρ^n) converges for :

(A) $\rho > 1$

(B) $-1 \leq \rho < 1$

(C) $\rho < -1$

(D) $-1 < \rho \leq 1$

6. The sequence $(n + (-1)^n)$ is :

(A) Convergent

(B) Divergent

(C) Oscillates finitely

(D) Oscillates infinitely

7. The sequence (a_n) defined by the recursion formula $a_{n+1} = \sqrt{3a_n}$, $a_1 = 1$ is :

(A) Monotonically decreasing

(B) Monotonically increasing

(C) Monotonically increasing and unbounded

(D) Monotonically increasing and bounded.

8. The sequence (a_n) where $a_n = 1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}$ converges to :

(A) 1

(B) 0

(C) $\frac{1}{2}$

(D) None of the above

9. If $\sum_{n=1}^{\infty} a_n$ is convergent, then for some finite number $\alpha \geq 1$ the series :

(i) $\sum_{n=1}^{\infty} a_n$

(ii) $\sum_{n=1}^{\infty} \alpha a_n$

(A) (i) is convergent (ii) is divergent

(B) (i) is divergent (ii) is convergent

(C) Both are convergent

(D) Both are divergent

10. The series $\sum_{n=1}^{\infty} \left\{ (n^3+1)^{\frac{1}{3}} - n \right\}^p$ is :

(A) Convergent for p = 1

(B) Divergent for p = 1

(C) Divergent for all values of p

(D) None of the above

11. The series $x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots$:

(A) Converges for $x > 1$

(B) Converges for $x < 1$

(C) Converges for $x = 1$

(D) Converges for all values of x

12. The series $1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} \cdots$ converges for :

(A) $p > 1$

(B) $p < 1$

(C) $p > 0$

(D) $p < 0$

13. The maximum value of $(x - 1)^2 e^x$ is attained at :

(A) e

(B) –e

(C) 1

(D) –1

14. $\lim_{x \to \frac{\pi}{2}} \frac{\log\left(x - \frac{\pi}{2}\right)}{\tan x} =$

(A) $\frac{\pi}{2}$

(B) $-\frac{\pi}{2}$

(C) 0

(D) 1

15. For which of the following functions Rolle's theorem fails ?

(A) $\sin x$; $x \in [-\pi, \pi]$

(B) $\frac{\sin x}{e^x}$; $x \in [0, \pi]$

(C) $|x|$; $x \in [-1, 1]$

(D) $x^3 - 4x$; $x \in [-2, 2]$

16. Which of the following functions doesn't have the Maclaurin's expansion :

(A) log x

(B) log(1 + x)

(C) log sec x

(D) log cos x

17. The angle between tangent and radius vector for the curve $r = a(1 + \cos\theta)$ at $\theta = \frac{\pi}{2}$ is :

(A) $\frac{\pi}{2}$

(B) $\frac{3\pi}{2}$

(C) $\frac{\pi}{4}$

(D) $\frac{3\pi}{4}$

18. The curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has the asymptotes :

(A) $y = \pm bx$

(B) $y = 0$

(C) $x = \pm by$

(D) $x = 0$

19. Which of the following is true for the functions

(i) $y = e^x$

(ii) $y = \log x$

(A) (i) is convex upwards (ii) is concave upwards

(B) (i) is concave upwards (ii) is convex upwards

(C) (i) is convex downwards (ii) is concave downwards

(D) (i) is concave downwards (ii) is convex downwards

20. Which of the following statements is true for the curve $x^4 + y^4 = 4a^2xy$?

(A) The curve is symmetrical w.r.t. $x + y = 0$

(B) The curve is symmetrical w.r.t. $x = 0$

(C) The curve is symmetrical w.r.t. $y = 0$

(D) The curve is symmetrical w.r.t. $x = y$

21. The functional sequence $f_n(x) = \frac{nx}{1 + n^2x^2}$ is :

(A) Uniformly convergent in [0, 1]

(B) Uniformly convergent in [–1, 1]

(C) Uniformly convergent in [–1, 0]

(D) Uniformly convergent in [1, 2]

22. The series $\sum_{n=1}^{\infty} f_n(x)$ is uniformly convergent on [a, b] if there exists a convergent series of positive numbers ΣM_n such that for all $x \in [a, b]$:

(A) $|f_n(x)| \leq M_n \ \forall \ n$

(B) $|f_n(x)| \geq M_n \ \forall \ n$

(C) $|f_n(x)| \leq \sqrt{M_n} \ \forall \ n$

(D) $|f_n(x)| \geq \sqrt{M_n} \ \forall \ n$

23. The limit of a sequence of functions is continuous if :

(A) Every function of the sequence is continuous

(B) The convergence is uniform

(C) Either (A) or (B)

(D) Both (A) and (B)

24. The function $f(x)=\begin{cases}3x-2 & , \quad x \le 0 \\ x+1 & , \quad x>0\end{cases}$ is :

(A) Continuous

(B) Discontinuous at x = –1

(C) Discontinuous at x = 2

(D) Discontinuous at the origin

25. For $u = \log(\tan x + \tan y)$, $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} =$

(A) 0

(B) 1

(C) 2

(D) 4

26. Leibnitz theorem is used to find :

(A) n^{th} derivative of sum of two functions

(B) n^{th} derivative of subtraction of two functions

(C) n^{th} derivative of product of two functions

(D) None of the above

27. $\lim_{x \to 0} (1+x)^{\frac{1}{x}} =$

(A) 0

(B) 1

(C) $\frac{1}{e}$

(D) e

28. $\int \frac{dx}{1-4\cos^2 x} =$

(A) $\frac{1}{2\sqrt{3}} \log \frac{\tan x + \sqrt{3}}{\tan x - \sqrt{3}}$

(B) $\frac{1}{2\sqrt{3}} \log \frac{\tan x - \sqrt{3}}{\tan x + \sqrt{3}}$

(C) $\frac{1}{2\sqrt{3}} \log \frac{\tan x + 2\sqrt{3}}{\tan x - 2\sqrt{3}}$

(D) $\frac{1}{2\sqrt{3}} \log \frac{\tan x - 2\sqrt{3}}{\tan x + 2\sqrt{3}}$

29. $\int \frac{dx}{(x+1)\sqrt{x+2}} =$

(A) $\log \frac{\sqrt{x+2}}{\sqrt{x+2}} - 1$

(B) $\log \frac{\sqrt{x+2}}{\sqrt{x+2}} + 1$

(C) $\log \frac{\sqrt{x+2}+1}{\sqrt{x+2}-1}$

(D) $\log \frac{\sqrt{x+2}-1}{\sqrt{x+2}+1}$

30. If $f(m, n) = \int_0^{\pi/2} \cos^m x \cos nx \, dx$ then :

(A) $f(m, n) = \frac{n}{m+n} f(m+1, n+1)$

(B) $f(m, n) = \frac{m}{m+n} f(m+1, n+1)$

(C) $f(m, n) = \frac{n}{m+n} f(m-1, n-1)$

(D) $f(m, n) = \frac{m}{m+n} f(m-1, n-1)$

31. For integer n, the Bessel functions $J_n(x)$ and $J_{-n}(x)$ are connected with the relation :

(A) $J_{-n}(x) = -J_n(x)$

(B) $J_{-n}(x) = (-1)^n J_n(x)$

(C) $J_{-n}(x) = -J_{-n}(x)$

(D) $J_{-n}(x) = (-1)^n J_{-n}(x)$

32. $\int J_n(x)\,dx =$

(A) $-J_2(x) - \frac{2}{x} J_2(x)$

(B) $J_2(x) - \frac{2}{x} J_2(x)$

(C) $-J_2(x) + \frac{2}{x} J_2(x)$

(D) None of the above

33. The Legendre polynomial for $m \neq n$,

$\int_{-1}^{1} P_m(x)\, P_n(x)\,dx =$

(A) –1

(B) 1

(C) 0

(D) None of the above

34. Which of the following sets is a group under multiplication modulo 8 ?

(A) {1, 3, 5, 7}

(B) {1, 2, 3, 4}

(C) {2, 4, 6, 8}

(D) {1, 2, 3, 4, 5, 6, 7}

35. What is the order of the dihedral group of square ?

(A) 2

(B) 4

(C) 6

(D) 8

36. Which of the following is not a group under multiplication ?

(A) $\mathbb{R} - \{0\}$

(B) $\mathbb{R}$

(C) {1, –1, i, –i}

(D) $\{1, \omega, \omega^2\}$

37. The identity of the set of positive rationals w.r.t. the binary operation $a * b = \frac{ab}{3}$ is :

(A) 0

(B) 1

(C) 3

(D) $\frac{1}{3}$

38. If G is a finite group of even order

(i) order of subgroup of G divides order of G

(ii) G has an element of order two.

(A) Only (i) is true

(B) Only (ii) is true

(C) Both are true

(D) Both are false

39. If H and K are subgroups of a group G of finite order and $O(H) > \sqrt{O(G)}$, $O(K) > \sqrt{O(G)}$, then :

(A) $O(H \cap K) = \sqrt{O(G)}$

(B) $O(H \cap K) = 1$

(C) $O(H \cap K) < 1$

(D) $O(H \cap K) > 1$

40. If $H \subseteq G$ be a subgroup of a group G then which of the following is not true ?

(A) $Ha = H$ if $a \in H$

(B) $Ha = H$ if $a \in G$

(C) $Ha = Hb$ if $ab^{-1} \in H$

(D) $ab^{-1} \in H$ if $Ha = Hb$

41. Which of the following statements is true ?

(A) Any finite group of order n is cyclic if it has an element of order n.

(B) Every cyclic group need not be abelian.

(C) The group of integers is not cyclic.

(D) None of the above

42. If M is a set of all matrices of the type $\begin{pmatrix} a & b \\ -\bar{b} & \bar{a} \end{pmatrix}$ where bars denote the complex conjugates, then M is a :

(A) Division ring

(B) Ring but not a division ring

(C) Field

(D) Not a ring

43. If R is a ring (i) $C = \{x \in R : xy = yx, \forall y \in R\}$, (ii) $N(a) = \{r \in R : ar = ra\}$ then :

(A) C is a subring, N(a) is not

(B) N(a) is a subring, C is not

(C) Both are subrings

(D) None is a subring

44. In a ring M of matrices of order 2 over integers the set $S = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} : a, b \in \mathbb{Z} \right\}$ is a :

(A) Right ideal

(B) Left ideal

(C) Two sided ideal

(D) None of the above

45. If R[x] denotes a polynomial ring over R then which of the following statements is true ?

(A) R is commutative if R[x] is commutative

(B) R[x] is commutative if R is commutative

(C) R has no proper zero divisors if R[x] has no proper zero divisors

(D) All the above

46. The union of two subgroups N_1 and N_2 of a group G is normal if :

(A) $N_1 \cap N_2 = \varphi$

(B) $N_1 \cap N_2 \neq \varphi$

(C) $N_1 \cup N_2 \neq \varphi$

(D) None of the above

47. Which of the following is true for the groups $G_1 = \{1, -1, i, -i\}$ and $G_2 = (\mathbb{Z}, +)$?

(A) All subgroups of G_1 are normal

(B) All the subgroups of G_2 are normal

(C) Both (A) and (B)

(D) None of the above

48. If N is a normal subgroup of a group G then :

(A) $O\left(\frac{G}{N}\right) = O(G) - O(N)$

(B) $O\left(\frac{G}{N}\right) = O(G) + O(N)$

(C) $O\left(\frac{G}{N}\right) = O(G) . O(N)$

(D) $O\left(\frac{G}{N}\right) = \frac{O(G)}{O(N)}$

49. $\frac{dx}{dy} + Qx = Px^n$ is a Bernoulli's differential equation where P and Q are :

(A) Functions of x only

(B) Functions of x or constants

(C) Constants only

(D) Functions of y or constants

50. $\frac{1}{D^2 + 4}\cos 2x =$

(A) $\frac{x}{4}\sin 2x$

(B) $\frac{x}{-4}\sin 2x$

(C) $\frac{x}{4}\cos 2x$

(D) $\frac{x}{-4}\cos 2x$

51. If M and N are homogenous in Mdx + Ndy = 0 then the integrating factor is :

(A) Mx + Ny

(B) Mx – Ny

(C) $\frac{1}{Mx + Ny}$

(D) $\frac{1}{Mx - Ny}$

52. $\frac{1}{f(D)} e^{-x} V =$

(A) $e^{-x} \frac{1}{f(D)} V$

(B) $e^{-x+1} \frac{1}{f(D)} V$

(C) $e^{-x} \frac{1}{f(D+1)} V$

(D) $e^{-x} \frac{1}{f(D-1)} V$

53. The solution of the ordinary differential equation $y^2 \log y = xyp + p^2$ is :

(A) $\log x = cy + c^2$

(B) $\log y = cx + c^2$

(C) $\log y^2 = cx + c^2$

(D) $\log x^2 = cy + c^2$

54. The solution of the ordinary differential equation $p^2 = y^2 + 1$ is :

(A) $y = \sin(c \pm x)$

(B) $y = \cos(c \pm x)$

(C) $y = \sinh(c \pm x)$

(D) $y = \cosh(c \pm x)$

55. What is the singular solution of the differential equation $2px + y^2p^3$?

(A) $y = cx + \frac{c^3}{8}$

(B) $y = cx - \frac{c^3}{8}$

(C) $y^2 = cx - \frac{c^3}{8}$

(D) $y^2 = cx + \frac{c^3}{8}$

56. The singular solution of the ordinary differential equation $y = xp + \frac{a}{p}$ is :

(A) $y^2 = 4ax$

(B) $y^2 = -4ax$

(C) $x^2 = 4ay$

(D) $x^2 = -4ay$

57. The partial differential equation by eliminating the arbitrary constants of the equation $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ is :

(A) $z = px + qy$

(B) $2z = px + qy$

(C) $2z = px - qy$

(D) $z = px - qy$

58. What is the general solution of the partial differential equation $\frac{y^2z}{x}p + xzq = y^2$.

(A) $\varphi(x^3 - y^3, x^3 - z^3) = 0$

(B) $\varphi(x^3 + y^3, x^3 + z^3) = 0$

(C) $\varphi(x^3 - y^3, x^2 - z^2) = 0$

(D) $\varphi(x^3 + y^3, x^2 + z^2) = 0$

59. The complete solution of the partial differential equation $\sqrt{p} + \sqrt{q} = 1$ is :

(A) $z = -ax + \left(1 - \sqrt{a}\right)^2 y + c$

(B) $z = ax - \left(1 - \sqrt{a}\right)^2 y + c$

(C) $z = -ax - \left(1 - \sqrt{a}\right)^2 y + c$

(D) $z = ax + \left(1 - \sqrt{a}\right)^2 y + c$

60. What is the solution of the partial differential equation $x + qy = pq$?

(A) $az = \frac{1}{2}(y + ax)^2 + b$

(B) $az = \frac{1}{2}(y - ax)^2 + b$

(C) $az = (y + ax)^2 + b$

(D) $az = (y - ax)^2 + b$

ROUGH WORK

ROUGH WORK

Sr. No. 1299

ENTRANCE TEST-2020

SCHOOL OF PHYSICAL & MATHEMATICAL SCIENCES

MATHEMATICS

Total Questions : 60
Time Allowed : 70 Minutes

Question Booklet Series **C**

Roll No. : | | | | | | |

Instructions for Candidates :

1. Write your Entrance Test Roll Number in the space provided at the top of this page of Question Booklet and fill up the necessary information in the spaces provided on the OMR Answer Sheet.
2. OMR Answer Sheet has an Original Copy and a Candidate's Copy glued beneath it at the top. While making entries in the Original Copy, candidate should ensure that the two copies are aligned properly so that the entries made in the Original Copy against each item are exactly copied in the Candidate's Copy.
3. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
4. Choose the correct / most appropriate response for each question among the options A, B, C and D and darken the circle of the appropriate response completely. The incomplete darkened circle is not correctly read by the OMR Scanner and no complaint to this effect shall be entertained.
5. Use only blue/black ball point pen to darken the circle of correct/most appropriate response. In no case gel/ink pen or pencil should be used.
6. Do not darken more than one circle of options for any question. A question with more than one darkened response shall be considered wrong.
7. There will be **'Negative Marking'** for wrong answers. Each wrong answer will lead to the deduction of 0.25 marks from the total score of the candidate.
8. Only those candidates who would obtain positive score in Entrance Test Examination shall be eligible for admission.
9. Do not make any stray mark on the OMR sheet.
10. Calculators and mobiles shall not be permitted inside the examination hall.
11. Rough work, if any, should be done on the blank sheets provided with the question booklet.
12. OMR Answer Sheet must be handled carefully and it should not be folded or mutilated in which case it will not be evaluated.
13. Ensure that your OMR Answer Sheet has been signed by the Invigilator and the candidate himself/ herself.
14. At the end of the examination, hand over the OMR Answer Sheet to the invigilator who will first tear off the original OMR sheet in presence of the Candidate and hand over the Candidate's Copy to the candidate.

1. In S_n the number of distinct cycles of length $r \leq n$ is :

(A) $\frac{1}{r}\frac{n!}{(n-r)!}$

(B) $r\frac{n!}{(n-r)!}$

(C) $n\frac{n!}{(n-r)!}$

(D) None

2. The identity of the group $\frac{G}{N}$ is :

(A) G

(B) $\frac{1}{G}$

(C) N

(D) $\frac{1}{N}$

3. Which of the following is not a ring (w.r.t. usual operations) ?

(A) $\mathbb{Z}[i] = \{a + ib : a, b \in \mathbb{Z}\}$

(B) $R = \{a + b\sqrt{2} + c\sqrt{3} + d\sqrt{2} : a, b, c, d \in \mathbb{Q}$

(C) $\mathbb{Q}[\sqrt{p}] = \{a + b\sqrt{p} : a, b \in \mathbb{Q}\}$ for prime p

(D) $R = \{a : a \in \mathbb{Q}^*\}$

4. If $R = \{f : [a, b] \to \mathbb{R},; f$ is continuous$\}$, then :

(A) R is not a ring

(B) R is a ring without unity

(C) R is a ring but not commutative

(D) R is a commutative ring with unity

5. $G_n = \left\{e^{\frac{2i\pi r}{n}} : r = 0,1,2,3,....,n-1\right\}$ is a cyclic group with generator :

(A) 1

(B) e

(C) $e^{\frac{2i\pi}{n}}$

(D) $e^{2in\pi}$

6. The symmetric group S_3 is :

(A) Abelian

(B) Infinite

(C) Non-Abelian

(D) Cyclic

7. Which of the following is a homomorphism from $(\mathbb{Z}, +)$ to $(\mathbb{R}, +)$?

(A) $f(a) = a^2$

(B) $f(a) = 2a$

(C) $f(a) = \frac{2}{a}$

(D) $f(a) = a^3$

8. Let M be the ring of all 2×2 matrices, then $S = \left\{\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} : a, b \in \mathbb{Z}\right\}$ is :

(A) Left ideal but not right ideal

(B) Right ideal but not left ideal

(C) Two sided ideal

(D) None of the above

9. Which of the following is true ?

(A) Ring of order 7 is non-commutative

(B) Ring with unity of order 25 is non-commutative

(C) Ring with unity of order 9 is commutative

(D) Ring of order 49 is always commutative

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10. Which of the following is not true ?
(A) Every integral domain is a field
(B) Every field is not an integral domain
(C) Every finite integral domain is a field
(D) Every finite integral domain is not a field

11. If R is a ring and R[x] the polynomial ring. Then :
(A) If R is commutative then R[x] is commutative
(B) If R has no proper zero divisors then R[x] has no proper zero divisors
(C) If R is an integral domain the R[x] is an integral domain
(D) If R is a field then R[x] is a field

12. For the Bessel function J, $[x^n J_n(x)]' =$
(A) $x^n J_{n-1}(x)$
(B) $x^{n-1} J_{n-1}(x)$
(C) $x^n J_n(x)$
(D) $nx^{n-1} J_{n-1}(x)$

13. The smallest positive root of $J_0(x) = 0$ lies in the interval :
(A) $(\sqrt{2}, 2)$
(B) (0, 1)
(C) $(3, \sqrt{10})$
(D) $(2, \sqrt{8})$

14. For the Legendre function, $P_n(-x) =$
(A) $-P_n(x)$
(B) $(-1)^n P_n(x)$
(C) $P_n(x)$
(D) $(-1)^{n+1} P_n(x)$

15. The order of the Legendre differential equation is :
(A) 1
(B) 2
(C) 3
(D) 4

16. Which of the following statements is not true ?
(A) Every infinite set has a bijection with its proper subset
(B) Every finite set can be put in one-one, onto correspondence with the set {1, 2, 3, ...n}
(C) Every infinite is equivalent to the set of integers
(D) Every infinite set contains a countable set

17. Which of the following functions is not uniformly continuous ?
(A) $f(x) = x^2$ in [1, 2]
(B) $f(x) = \sqrt{x}$ in [0, 2]
(C) $f(x) = x$ in [0, 1]
(D) $f(x) = \frac{1}{x}$ in (0, 1)

18. The number of limit points of the set $\left\{\frac{1}{n}; n = 1,2,3,.....\right\}$ is :
(A) 1
(B) 2
(C) 0
(D) ∞

19. Which of the following sequences is not convergent ?
(A) $\left(\frac{1}{n}\right)$
(B) $\left(\frac{1}{n^2}\right)$
(C) $\left(\frac{n}{n+1}\right)$
(D) $\left(\frac{n^2}{n+1}\right)$

20. The series $\frac{1}{(\log 2)^p} + \frac{1}{(\log 3)^p} + ... \frac{1}{(\log n)^p} + ...$
(A) Converges for $p > 0$
(B) Converges for $p < 0$
(C) Diverges for $p > 0$
(D) Oscillates

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21. Which among the following series is not convergent ?

(A) $\frac{p^n}{n!}, p>0$

(B) $\frac{p^n}{n^n}, p>0$

(C) $\frac{\sqrt{n}x^n}{\sqrt{n^2+1}}, x \geq 1$

(D) $\frac{1.2.3....n}{7.10....(3n+4)}$

22. Which of the following statements is not true ?

(A) Every bounded sequence is convergent

(B) Every convergent sequence is bounded

(C) Every bounded monotonic sequence is convergent

(D) Every bounded sequence with unique limit point is convergent

23. For the series (i) $\sum \frac{(-1)^{n+1}}{\sqrt{n}}$ (ii) $\sum \frac{(-1)^{n+1}}{3n-2}$:

(A) Both are absolutely convergent

(B) Only (i) is absolutely convergent

(C) Only (ii) is absolutely convergent

(D) Both are conditionally convergent

24. The function $f(x) = \begin{cases} \frac{\sin 2x}{x} & \text{when } x \neq 0 \\ 1 & \text{when } x = 0 \end{cases}$ has :

(A) Removable discontinuity at origin

(B) Essential discontinuity

(C) No discontinuity

(D) None of the above

25. Which of the following sets is not countable ?

(A) $\mathbb{Z} \times \mathbb{Z}$

(B) $\mathbb{Q}$

(C) $\{x : 0 < x < 1\}$

(D) $\left\{\frac{1}{n} : n = 1,2,3,.....\right\}$

26. The set $S = \{x : x \in \mathbb{Q} \text{ and } x^2 < 2\}$:

(A) Is a bounded above subset and supremum exists in $\mathbb{Q}$

(B) Is not bounded above subset of $\mathbb{Q}$

(C) Is a bounded above and has not a supremum in $\mathbb{Q}$

(D) None

27. If (a_n) is a Cauchy sequence in $\mathbb{R}$, then :

(A) (a_n) is bounded but not convergent

(B) (a_n) is convergent but not bounded

(C) (a_n) is convergent as well as bounded

(D) Nothing can be said

28. The sequence of functions $f_n(x) = e^{-nx}, x \in [1, 3]$ is :

(A) Point-wise convergent but not uniformly convergent

(B) Uniformly convergent

(C) Not convergent

(D) None

29. The series $\sum_{n=1}^{\infty} \frac{(-1)^n n^{500}}{(1.0001)^n}$:

(A) Converges absolutely

(B) Converges conditionally not absolutely

(C) Diverges to $-\infty$

(D) Diverges to ∞

30. According to root test the positive term series $\sum a_n$ converges if $l < 1$ where :

(A) $l = \lim_{n\to\infty} (a_n)^n$

(B) $l = \lim_{n\to\infty} (a_n)^{\frac{1}{n}}$

(C) $l = \lim_{n\to\infty} (na_n)^n$

(D) $l = \lim_{n\to\infty} (na_n)^{\frac{1}{n}}$

31. $r = ae^{m\theta}$ represents a :

(A) Circle

(B) Parabola

(C) Cardioid

(D) Spiral

32. Which of the following functions does not have a Taylor's expansion in the interval $[-1,1]$?

(A) $|x|$

(B) $\sin x$

(C) $\cos x$

(D) e^x

33. If $f(x) = x^2 - 6x + 8$ in the interval $[2,4]$. Then $f'(x) = 0$, where $x =$

(A) 2

(B) 4

(C) 3

(D) 1

34. Leibnitz's theorem is used to find :

(A) n^{th} derivative of trigonometric function

(B) n^{th} derivative of exponential functions

(C) n^{th} derivative of quotient of two functions

(D) n^{th} derivative of product of two functions

35. If $y = 100^x$. Then the n^{th} derivative $y_n =$

(A) 100^x

(B) $100^x \times 100^n$

(C) $100^x \times 2^n$

(D) $100^x \times 2$

36. The degree of the homogeneous function $\frac{x^2+y^2}{x^2-y^2}\cos\frac{y}{x}$ is :

(A) 0

(B) 1

(C) 2

(D) -1

37. For the functions (i) $y = e^x$ (ii) $y = \log x$:

(A) Both are concave

(B) Both are convex

(C) (i) is concave (ii) is convex

(D) (i) is convex (ii) is concave

38. Which of the following statement is true ?

(A) Angle between two curves is the angle between their tangents

(B) Angle between two curves is the angle between their normals

(C) Angle between two curves is the angle between tangent of one curve and normal of other

(D) None of the above statements is true

39. The maximum value of the function $f(x) = \frac{\log x}{x}$ is :

(A) e

(B) e^2

(C) log e

(D) $\frac{1}{e}$

40. $\lim_{\theta\to 0} \frac{\sin\theta - \theta\cos\theta}{\sin\theta - \theta} =$

(A) 1

(B) -1

(C) -2

(D) 2

41. The curvature of an ellipse is maximum at :
(A) Ends of major axis
(B) Ends of minor axis
(C) Ends of the line through focus
(D) None

42. The number of asymptotes to the curve $\left(\frac{y}{x}\right)^2 = \frac{4a}{x}$ is :
(A) 1
(B) 2
(C) 3
(D) 0

43. The length of polar sub-tangent to the curve $f(r, \theta) = 0$ is :
(A) $r^2 \frac{d\theta}{dr}$
(B) $r^2 \frac{dr}{d\theta}$
(C) $\frac{1}{r^2} \frac{d\theta}{dr}$
(D) $\frac{1}{r^2} \frac{dr}{d\theta}$

44. $\int_0^{\pi} \frac{\sin nx}{\sin x} dx =$
(A) 0 if n is even
(B) π if n is odd
(C) Both (A) and (B) true
(D) None

45. $\int \frac{\sqrt{x}}{x+1} =$
(A) $2\sqrt{x} - 2\tan^{-1}\sqrt{x}$
(B) $2\sqrt{x} - 2\sin^{-1}\sqrt{x}$
(C) $2\sqrt{x} - 2\cos^{-1}\sqrt{x}$
(D) $2\sqrt{x} - 2\cot^{-1}\sqrt{x}$

46. (i) $\int$ (ii) $\frac{dy}{dx}$
(A) (i) is linear (ii) is not
(B) (ii) is linear (i) is not
(C) Both are linear
(D) Both are not linear

47. $\int_0^{\frac{\pi}{2}} \sin^6 x dx =$
(A) $\frac{6\pi}{32}$
(B) $\frac{3\pi}{32}$
(C) $\frac{\pi}{32}$
(D) $\frac{5\pi}{32}$

48. The order and degree of the differential equation $\left\{1 + \left(\frac{d^2y}{dx^2}\right)^3\right\}^2 = 3\left(\frac{d^3y}{dx^3}\right)^2$ is :
(A) 3 and 2
(B) 2 and 3
(C) 3 and 6
(D) 6 and 6

49. The integrating factor of the differential equation $xdy = y(1 + xy)dy$ is :
(A) e^x
(B) e^{-x}
(C) x
(D) $-x$

50. The differential equation $Mdy + Ndx = 0$ is exact if :
(A) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
(B) $\frac{\partial N}{\partial y} = \frac{\partial M}{\partial x}$
(C) $\frac{\partial M}{\partial y} = -\frac{\partial N}{\partial x}$
(D) $\frac{\partial N}{\partial y} = -\frac{\partial M}{\partial x}$

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51. The solution of the differential equation $(1+x^2)\left(\frac{dy}{dx}-4x^2\cos^2 y\right) + x \sin 2y = 0$ with integrating factor $(1 + x^2)$ is :

(A) $(1 + x^2) \tan y = \frac{4x^3}{3}+3$

(B) $(1 + x^2) \sec y = \frac{4x^3}{3}+3$

(C) $(1 + x^2) \sin y = \frac{4x^3}{3}+3$

(D) $(1 + x^2) \cot y = \frac{4x^3}{3}+3$

52. The solution of the differential equation $p^2y + 2py = y$ is :

(A) $x^2 = 2 cxy + c^2$

(B) $y^2 = 2 cxy + c^2$

(C) $x^2 = 2 cy + c^2$

(D) $y^2 = 2 cx + c^2$

53. The solution of the differential equation $(x^2D^2 + 3xD)y = \frac{1}{x}$ is :

(A) $A + Bx^{-2} + x^{-1}$

(B) $A + Bx + x^{-1}$

(C) $A + Bx^{-2} + x$

(D) $A + Bx^2 + x^{-1}$

54. The number of constants in the solution of a partial differential equation depends on :

(A) Order of differential equation

(B) Degree of differential equation

(C) Number of independent variables

(D) Number of dependent variables

55. The differential equation $5\frac{\partial^2 z}{\partial x^2}+6\frac{\partial^2 z}{\partial y^2} = xy$ is classified as :

(A) Elliptical

(B) Parabolic

(C) Hyperbolic

(D) None

56. Which of the following is not a group ?

(A) $(\mathbb{Z}, +)$

(B) $(\mathbb{R}, +)$

(C) $(\mathbb{R}, \times)$

(D) $(\mathbb{Q}, +)$

57. If $a * b = \frac{ab}{2}$ $a, b \in \mathbb{Q}^+$, $\mathbb{Q}^+$ is set of positive rationals :

(A) 1

(B) 2

(C) $\frac{3}{2}$

(D) $\frac{1}{2}$

58. The order of i in the multiplicative group $\{-1, 1, -i, i\}$ is :

(A) 1

(B) 2

(C) 3

(D) 4

59. If $(\mathbb{Z}, *)$ is a group with $* b = a + b + 1, \forall a, b \in \mathbb{Z}$. Then inverse of an element a is :

(A) 0

(B) –2

(C) $-a - 2$

(D) $a + 2$

60. Let $S = \{1, 2, 3, 4, 5\}$ and $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 3 & 5 \end{pmatrix}$, then the number of orbits of S under the permutation f is :

(A) 1

(B) 2

(C) 3

(D) 4

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1. The locus of the middle points of a system of parallel chords of a parabola $y^2 = 4ax$ is :

(A) A straight line

(B) A circle

(C) An ellipse

(D) A parabola

2. The eccentricity of an ellipse whose latus rectum is equal to one half of its minor axis is :

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{1}{\sqrt{2}}$

3. If e and e* are the eccentricities of a hyperbola and a conjugate hyperbola, then :

(A) $e^2 + e^{*2} = 1$

(B) $e^2 + e^{*2} = e^2e^{*2}$

(C) $e^2 e^{*2} = 1$

(D) $e + e^* = ee^*$

4. If $ab - h^2 = 0$, then the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents :

(A) A parabola

(B) An ellipse

(C) A circle

(D) A hyperbola

5. Which of the following is the condition for the spheres $a(x^2+y^2+z^2)+2lx+2my+n=0$ and $b(x^2+y^2+z^2)=k^2$ to cut orthogonally ?

(A) $ap^2 = bk$

(B) $bp^2 = ak$

(C) $pk^2 = ab$

(D) $ak^2 = bp$

6. The equation $ax^2+by^2+cz^2+2ux+2vy+2wz+d=0$ will represent a cone if :

(A) $\frac{a^2}{u}+\frac{b^2}{v}+\frac{c^2}{w}=d$

(B) $au^2 + bv^2 + cw^2 = d$

(C) $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=d$

(D) $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=0$

7. The equation of the right circular cylinder whose axis is the z-axis and radius is 'a' is :

(A) $x^2 + y^2 = a^2$

(B) $x^2 + z^2 = a^2$

(C) $y^2 + z^2 = a^2$

(D) $x^2 + y^2 + z^2 = a^2$

8. The equation of the tangent planes to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ which are parallel to $lx + my + nz = 0$ are :

(A) $lx + my + nz = \pm\sqrt{a\ell^2 + bm^2 + cn^2}$

(B) $lx + my + nz = \pm\sqrt{a^2\ell^2 + b^2m^2 + c^2n^2}$

(C) $lx + my + nz = \pm\sqrt{\frac{\ell^2}{a}+\frac{m^2}{b}+\frac{n^2}{c}}$

(D) $lx + my + nz = \pm\sqrt{\frac{\ell^2}{a^2}+\frac{m^2}{b^2}+\frac{n^2}{c^2}}$

9. If $A=\begin{bmatrix}2&1&1\\0&1&0\\1&1&2\end{bmatrix}$, then $A^4-5A^3+7A^2-2A+I$ (I is the unit matrix of order 3) is equal to :

(A) $\begin{bmatrix}3&1&1\\0&2&0\\1&1&3\end{bmatrix}$

(B) $\begin{bmatrix}2&1&1\\0&1&0\\1&1&2\end{bmatrix}$

(C) $\begin{bmatrix}8&5&5\\0&3&0\\5&5&8\end{bmatrix}$

(D) $\begin{bmatrix}5&4&4\\0&1&0\\1&1&3\end{bmatrix}$

10. The rank of the matrix $\begin{bmatrix}2&3&4&-1\\5&2&0&-1\\-4&5&12&-1\\0&0&0&0\end{bmatrix}$ is :

(A) 4
(B) 3
(C) 2
(D) 1

11. The necessary and sufficient condition for a matrix to be scalar is that its minimal equation is of degree :
(A) 4
(B) 3
(C) 2
(D) 1

12. Which of the following is true for any three square matrices P, Q, R each of order n ?
(A) Tr(PQR) = Tr(QRP)
(B) Tr(PRQ) = Tr(RQP)
(C) Tr(RQP) = Tr(RPQ)
(D) Tr(RPQ) = Tr(QRP)

13. Which of the following is true for a square matrix A of order n whose rank is less than n ?
(A) $|A| = 0$
(B) The columns of A are linearly dependent
(C) The rows of A are linearly dependent
(D) All of these

14. The value of 'a' for which the system of equations $x+2y+3z=ax$, $3x+y+2z=ay$, $2x+3y+z=az$ have non-zero solutions is :
(A) $a=2$
(B) $a=4$
(C) $a=6$
(D) $a=1$

15. The number of linearly indpendent solutions of the equation $x+y+z+t=1$ is :
(A) 2
(B) 3
(C) 4
(D) 5

16. Which of the following is not a normal vector ?

(A) $\begin{bmatrix}1\\i\\1-i\end{bmatrix}$

(B) $\begin{bmatrix}\frac{2}{3}\\\frac{2}{3}\\\frac{1}{3}\end{bmatrix}$

(C) $\begin{bmatrix}\sin\theta\\-\cos\theta\\0\end{bmatrix}$

(D) $\begin{bmatrix}\frac{1}{\sqrt{3}}\\-\frac{1}{\sqrt{3}}\\\frac{1}{\sqrt{3}}\end{bmatrix}$

17. Which of the following is the solution of the differential equation $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$?

(A) $\tan x = \frac{1}{2}(y^2 - 1) + ce^{-y^2}$

(B) $\tan y = \frac{1}{2}(x^2 - 1) + ce^{-x^2}$

(C) $\tan y = \frac{1}{2}(x^2 - 1)e^{-x^2} + c$

(D) $\tan x = \frac{1}{2}(y^2 - 1)e^{-y^2} + c$

18. The solution of the differential equation

$$xdy + ydx + \frac{xdy - ydx}{x^2 + y^2} = 0 \text{ is :}$$

(A) $x + y \tan^{-1} \frac{y}{x} = c$

(B) $y + x \tan^{-1} \frac{x}{y} = c$

(C) $xy + \tan^{-1} \frac{y}{x} = c$

(D) $xy + \tan^{-1} \frac{x}{y} = c$

19. The particular integral of the differential equation $(D^2 + 5D + 4)y = x^2 + 7x + 9$ is :

(A) $\frac{1}{4}\left(x^2 + \frac{9}{2}x\right)$

(B) $\frac{1}{4}\left(x^2 + \frac{23}{8}\right)$

(C) $\frac{1}{4}\left(x^2 + \frac{9}{2}x + \frac{8}{23}\right)$

(D) $\frac{1}{4}\left(x^2 + \frac{9}{2}x + \frac{23}{8}\right)$

20. Which of the following is the general solution of the differential equation $\frac{d^2y}{dx^2} + y = \text{cosec } x$?

(A) $y = c_1 \cos x + c_2 \sin x + \sin x \log \sin x$

(B) $y = c_1 \cos x + c_2 \sin x + \cos x \log \cos x - x \sin x$

(C) $y = c_1 \cos x + c_2 \sin x + \sin x \log \sin x - x \cos x$

(D) $y = c_1 \cos x + c_2 \sin x + \cos x \log \cos x - x \cos x$

21. Which of the following is not true ?

(A) Any subset of a countable set is countable

(B) Any subset of an uncountable set is uncountable

(C) If A and B are countable sets, then $A \cup B$ and $A \cap B$ are also countable

(D) Every infinite set contains a countable set

22. Which of the following is true for any sequence $\{a_n\}$ of real numbers ?

(A) If $\{a_n\}$ is monotonically increasing, then it is convergent.

(B) If $\{a_n\}$ is monotonically decreasing, then it is convergent.

(C) If $\{a_n\}$ is bounded, then it is convergent.

(D) If $\{a_n\}$ is a Cauchy sequence, then it is convergent.

23. For what value of x does the series $1 + x + x^2 + x^3 + \ldots$. oscillate infinitely ?

(A) $x = -1$

(B) $|x| < 1$

(C) $x \geq 1$

(D) $x < -1$

24. Which of the following series is not convergent ?

(A) $\sum_{n=1}^{\infty} \cot^{-1} n^2$

(B) $\sum_{n=1}^{\infty} \frac{n^p}{n!}$

(C) $\sum_{n=1}^{\infty} \frac{1^2.3^2.5^2 \ldots (2n-1)^2}{2^2.4^2.6^2 \ldots (2n)^2}$

(D) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\log n}$

25. The value of $\iint_R e^{\frac{y}{x}} dydx$, where $R = \{(x, y), 0 < x < 1, 0 < y < x^2\}$ is

(A) 1

(B) 2

(C) e

(D) $\frac{1}{2}$

26. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, then $div(r^n \vec{r}) =$

(A) $(n+1)r^n$

(B) $(n+2)r^{n+1}$

(C) $(n+3)r^{n+2}$

(D) $(n+3)r^n$

27. Which of the following is true for the function

$f(x, y) = \sqrt{|xy|}$?

(A) f(x, y) is differentiable at the origin

(B) f(x, y) is not differentiable at the origin

(C) The partial derivatives f_x, f_y are continuous at the origin

(D) None of the above

28. Which of the following is true for the function $f(x, y) = x^2 - 3xy^2 + 2y^4$?

(A) f(x, y) has a maximum value at the origin

(B) f(x, y) has a minimum value at the origin

(C) f(x, y) has neither a maximum value nor a minimum value at the origin

(D) None of the above

29. If f(x) is bounded and integrable on [a, b] and $f(x) \geq 0$, $\forall x \in [a, b]$, then :

(A) $\int_a^b f(x)\,dx \geq 0$ for $a \leq b$

(B) $\int_a^b f(x)\,dx \leq 0$ for $a \leq b$

(C) $\int_a^b f(x)\,dx \geq 0$ for $b \leq a$

(D) None of the above holds

30. Which of the following is/are necessary and sufficient condition(s) for a bounded function f to be integrable on [a, b] ?

(A) For any $\varepsilon > 0$, there exists a partition P of [a, b] such that $U(P, f) - L(P, f) < \varepsilon$.

(B) For any $\varepsilon > 0$, there exists a partition P of [a, b] and a number I lying between L(P, f) and U(P, f) such that :

$|U(P, f) - I| < \varepsilon$ and $|L(P, f) - I| < \varepsilon$.

(C) For any $\varepsilon > 0$, there exists a $\delta > 0$ such that if P, P* are any two partitions of [a, b] with mesh less than δ, then :

$|S(P, f) - S(P^*, f)| < \varepsilon$.

(D) All the above

31. Which of the following is the value of $\int_{-1}^{1} (x + |x|)dx$?

(A) 0

(B) −1

(C) 1

(D) 2

32. Which of the following is true for the function f defined on [0, 1] by :

f(x) = 0, when x is irrational or zero = 1/n, when x is any non-zero rational number m/n in its lowest form.

(A) f(x) is integrable on [0, 1] and the value of the integral is 0.

(B) f(x) is integrable on [0, 1] and the value of the integral is 1.

(C) f(x) is not integrable on [0, 1].

(D) None of the above

33. Which of the following is/are true for any group G of prime order ?

(A) G is cyclic

(B) G is Abelian

(C) G has exactly two subgroups

(D) All the above

34. Which of the following is not true ?

(A) Every group is a normal subgroup of itself.

(B) The center of every group is a normal subgroup of that group.

(C) Every subgroup of an Abelian group is normal.

(D) There is no non-Abelian group for which every subgroup is normal.

35. The exponential map from the group C of complex numbers with addition to the group C* of non-zero complex numbers with multiplication is a homomorphism with kernel :

(A) $\{0\}$

(B) $\{1\}$

(C) $\{2k\pi i, k \in Z\}$

(D) $\{k\pi i, k \in Z\}$

36. If P(S) is the power set of a non-empty set S with addition to be the symmetric difference of sets and multiplication to be intersection, then P(S) is :

(A) A division ring

(B) A Boolean ring

(C) A field

(D) None of these

37. If S and S* are subspaces of a vector space X over the field F, which of the following is/are subspaces of X over F ?

(A) $S + S^*$

(B) $S \cap S^*$

(C) $L(S \cup S^*)$

(D) All the above

38. Which of the following is a basis for the vector space C of complex numbers over itself ?

(A) $\{0\}$

(B) $\{1\}$

(C) $\{1, i\}$

(D) $\{i\}$

39. If T is a homomorphism from X to Y, where X and Y are vector spaces over the same field F, with kernel K, then T is an isomorphism if and only if K =

(A) $\{0\}$

(B) Φ

(C) X

(D) Y

40. If X and Y are finite-dimensional vector spaces over the same field F and f is a homomorphism from X onto Y with kernel K, then :

(A) $\dim(X) = \dim(Y)$

(B) $\dim(X) = \dim(Y) + \dim(K)$

(C) $\dim(Y) = \dim(X) + \dim(K)$

(D) $\dim(Y) = \dim(X)/\dim(K)$

41. $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} =$

(A) 0

(B) 1

(C) 2

(D) -1

42. Which of the following is true for the function :

$$f(x) = e^{\frac{1}{(x-2)^2}}, x \neq 2$$

$$= 0, x = 2 ?$$

(A) f(x) is continuous at x = 2

(B) f(x) has a discontinuity of first kind at x = 2

(C) f(x) has a discontinuity of second kind at x = 2

(D) None of the above

43. If $f(x) = x^{n-1} + e^{2x}$, then $f^n(0) =$

(A) $(n-1)! + 2^n$

(B) 2^n

(C) $(n-1)! + 2^{n-1}$

(D) $2^n + 1$

44. The coefficient of $(x-a)^2$, when $\log \sin x$ is expanded as a series in $(x-a)$, is :

(A) $\log \sin a$

(B) $\cot a$

(C) $\frac{1}{2}\operatorname{cosec}^2 a$

(D) $-\frac{1}{2}\operatorname{cosec}^2 a$

45. Which of the following is the pedal equation of the parabola $\frac{2a}{r} = 1 - \cos\theta$?

(A) $p^2 = ar$

(B) $r^2 = ap$

(C) $a^2 = pr$

(D) $p = ar$

46. If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(A) $\frac{1}{2}\cos u$

(B) $\frac{1}{2}\cot u$

(C) $-\frac{1}{2}\cot u$

(D) $-\frac{1}{2}\cos u$

47. What is the radius of curvature of the circle $r = a\cos\theta$?

(A) a

(B) $\frac{a}{2}$

(C) 2a

(D) a^2

48. The equation of the hyperbola having $x + y - 1 = 0$ and $x - y + 2 = 0$ as its asymptotes and passing through the origin is :

(A) $x + y - 1 = 2(x - y + 2)$

(B) $(x + y - 1)(x - y + 2) + 2 = 0$

(C) $(x + y - 1)(x - y + 2) = 0$

(D) $(x + y - 1)(x - y + 2) - 2 = 0$

49. The equation $z\bar{z} + (1+i)z + (1-i)\bar{z} = 0$ represents :

(A) A straight line

(B) A circle

(C) An ellipse

(D) None of these

50. If $\sin\phi = i\tan\theta$, then $\cos\theta + i\sin\theta =$

(A) $\tan\frac{\phi}{2}$

(B) $\tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$

(C) $\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$

(D) $\tan\phi$

51. The principal value of i^i is :

(A) $e^{\frac{\pi}{2}}$

(B) $e^{-\frac{\pi}{2}}$

(C) e^{π}

(D) $e^{-\pi}$

52. Which of the following is the imaginary part of $\sin^{-1}(\cos\theta + i\sin\theta)$, $0 < \theta < \frac{\pi}{2}$?

(A) $\cos^{-1}\sqrt{\sin\theta}$

(B) $\sin^{-1}\sqrt{\cos\theta}$

(C) $\log(\sqrt{\sin\theta} + \sqrt{1+\sin\theta})$

(D) $\log(\sqrt{1+\sin\theta} - \sqrt{\sin\theta})$

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53. When $x^4 - 3x^3 + 4x^2 - 6x + 7$ is divided by $x - 1$, then the remainder is :
(A) 7
(B) 6
(C) 5
(D) 3

54. The equation which increases the roots of the equation $x^3 + 6x^2 + 7x + 2 = 0$ by 2 is :
(A) $x^3 - 5x + 4 = 0$
(B) $x^3 - 4x + 5 = 0$
(C) $x^3 - 6x + 7 = 0$
(D) $x^3 - 7x + 6 = 0$

55. The equation whose roots are 2 times the roots of the equation $x^7 - 5x^4 + 13x^2 - 11 = 0$ is :
(A) $x^7 - 10x^4 + 52x^2 - 88 = 0$
(B) $x^7 - 20x^4 + 52x^2 - 88 = 0$
(C) $x^7 - 40x^4 + 208x^2 - 352 = 0$
(D) $x^7 - 40x^4 + 416x^2 - 1408 = 0$

56. For what value of k the roots of the equation $x^3 - 6x + 11x + k = 0$ are in A.P. ?
(A) $k = 4$
(B) $k = 5$
(C) $k = -6$
(D) $k = 6$

57. If $x + y + z = 1$, $x^2 + y^2 + z^2 = 2$, $x^3 + y^3 + z^3 = 3$, then $x^5 + y^5 + z^5 =$
(A) 4
(B) 5
(C) 6
(D) 7

58. If α, β, γ are the roots of the equation $x^3 + 2x^2 - 3x - 1 = 0$, then the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3} + \frac{1}{\gamma^3}$ is :
(A) 42
(B) –42
(C) 24
(D) –36

59. If $G^2 + 4H^3 < 0$, then the cubic equation $x^3 + 3Hx + G = 0$ has :
(A) One real and two conjugate complex roots
(B) All the three roots real and two of them are equal
(C) All the three roots real and distinct
(D) No real roots

60. Which of the following is true for any zero 'a' of the polynomial ?
$z^5 + z^4 + z^3 + z^2 + z + 1$?
(A) $|a| < 1$
(B) $|a| = 1$
(C) $|a| > 1$
(D) None of these

1. The value of $\int_0^{\frac{\pi}{6}} \sin^8 3\theta d\theta$ is :

(A) $\frac{3\pi}{16}$

(B) $\frac{5\pi}{32}$

(C) $\frac{64}{35}$

(D) $\frac{35\pi}{768}$

2. The order and the degree of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = k\frac{d^2y}{dx^2}$$

are respectively :

(A) 2 and 1

(B) 1 and 2

(C) 2 and 2

(D) 1 and 1

3. The solution of the differential equation $ydx - xdy + 3x^2y^2e^{x^3}dx = 0$ is :

(A) $x + ye^{x^3} = cy$

(B) $y + xe^{x^3} = cx$

(C) $x - ye^{x^3} = cy$

(D) $y - xe^{x^3} = cx$

4. The particular integral of the differential equation

$$(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}$$

is :

(A) $-\frac{1}{120}(e^{-2x} + e^{-3x})$

(B) $-\frac{1}{120}(e^{-2x} + 3e^{-3x})$

(C) $-\frac{1}{120}(2e^{-2x} + e^{-3x})$

(D) $\frac{1}{120}(2e^{-2x} + e^{-3x})$

5. If P and Q are non-singular square matrices of the same order, then adj(PQ) =

(A) adj(P).adj(Q)

(B) adj(Q).adj(P)

(C) |PQ|.I

(D) |PQ|PQ

6. Which of the following is not true for any non-singular matrix M with transpose M′ and inverse M^{-1} ?

(A) $|M'| = |M|$

(B) $M^{-1} = M'$

(C) $(M^{-1})' = (M')^{-1}$

(D) $Tr(M') = Tr(M)$

7. If $P = \begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}$, then P^5 =

(A) 3P

(B) 9P

(C) 27P

(D) 81P

8. The rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ is :

(A) 4

(B) 3

(C) 5

(D) 2

9. The number of linearly independent solutions of the equation $x + y + z = 1$ is :

(A) 4

(B) 3

(C) 1

(D) 2

10. Which of the following is not true ?

(A) The columns of a non-singular matrix are linearly independent

(B) The rows of a matrix A of order $m \times n$ are linearly dependent iff rank of A is less than m

(C) A square matrix A of order n is non-singular iff its rank is less than n

(D) For any matrices of suitable order, $\text{rank}(ABC) \leq \text{rank}(AB)$

11. For what values of m and n, the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + mz = n$ have no solution ?

(A) $m = 3, n = 10$

(B) $m \neq 3, n = 10$

(C) $m = 3, n \neq 10$

(D) $m \neq 3, n \neq 10$

12. For any orthogonal matrix P :

(A) $P^{-1} = P$

(B) $PP' = P$

(C) $PP' = P'$

(D) $P^{-1} = P'$

13. For what values of m does the equation $2x^3 - 9x^2 + 12x + m = 0$ have two equal roots ?

(A) $m = -1, -4$

(B) $m = -4, -3$

(C) $m = -4, -5$

(D) $m = -1, -3$

14. If α, β, γ are the roots of the equation $x^3 - 6x^2 + 12x - 8 = 0$, then the equation whose roots are $\alpha - 2, \beta - 2, \gamma - 2$ is :

(A) $x^3 + 6x^2 - 12x + 8 = 0$

(B) $x^3 - 8x^2 + 12x - 6 = 0$

(C) $x^3 = 0$

(D) $x^3 + 6 = 0$

15. If α, β, γ are the roots of the equation $x^3 - x^2 + 8x - 6 = 0$, then the equation whose roots are $\alpha^2, \beta^2, \gamma^2$ is :

(A) $x^3 + 15x^2 + 52x - 36 = 0$

(B) $x^3 - 15x^2 - 52x + 36 = 0$

(C) $x^3 - 15x^2 + 52x - 36 = 0$

(D) $x^3 + 15x^2 - 52x + 36 = 0$

16. The condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ may be in G.P. is :

(A) $pq = r$

(B) $pr = q$

(C) $2p^3 - 9pq + 27r = 0$

(D) $p^3r = q^3$

17. If α, β, γ are the roots of the equation $x^3 + 2x^2 - 3x - 1 = 0$, then $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} =$

(A) 41

(B) -42

(C) -52

(D) 23

18. The number of imaginary roots of the equation $x^7 - 3x^4 + 2x^3 - 1 = 0$ is at least :

(A) 2

(B) 3

(C) 4

(D) 5

19. Which of the following is true for any root 'a' of the equation $z^4 + z^2 + 1 = 0$?

(A) $|a| = 1$

(B) $|a| < 1$

(C) $|a| > 1$

(D) None of these

20. The sum of the cubes of the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$ is :

(A) 6

(B) 11

(C) 36

(D) 121

21. Which of the following is not true ?

(A) If A and B are countable sets, then $A \cup B$ is also a countable set

(B) If A is a countable set, then every subset of A is also countable

(C) If A is an uncountable set, then every subset of A is also uncountable

(D) If A and B are countable sets, then so is their Cartesian product

22. Which of the following is not true for any two bounded sequences $\{a_n\}$ and $\{b_n\}$ of non-negative real numbers ?

(A) $\underline{\lim}(a_n b_n) \le \underline{\lim}(a_n).\underline{\lim}(b_n)$

(B) $\underline{\lim} a_n.\overline{\lim} b_n \le \overline{\lim}(a_n b_n)$

(C) $\overline{\lim}(a_n b_n) \le \overline{\lim} a_n..\overline{\lim} b_n$

(D) $\underline{\lim}(a_n b_n) \le \underline{\lim} a_n.\overline{\lim} b_n$

23. Which of the following series is not convergent ?

(A) $\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$

(B) $\sum_{n=0}^{\infty} \{(n^3+1)^{\frac{1}{3}} - n\}$

(C) $\sum_{n=0}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n}$

(D) $\sum_{n=1}^{\infty} \sin\frac{1}{n^2}$

24. The series $\sum_{n=1}^{\infty} \frac{1^2.3^2.....(2n-1)^2}{2^2.4^2.....(2n)^2} x^{n-1}, x > 0$ converges for :

(A) $x > 1$

(B) $x = 1$

(C) $x < 1$

(D) None of these

25. The value of $\int_0^4 [x]dx$ is :

(A) 4

(B) 8

(C) 6

(D) 17

26. Which of the following is true for the function

$f(x) = x$, when x is rational

$= -x$, when x is irrational

on [0, 1] ?

(A) f is integrable, but $|f|$ is not integrable

(B) $|f|$ is integrable, but f is not integrable

(C) f and $|f|$ are both integrable

(D) Neither f nor $|f|$ is integrable

27. Which of the following is true for any two bounded and integrable functions f and g on [a, b] ?

(A) f+g is integrable

(B) f–g is integrable

(C) f.g is integrable

(D) $\frac{f}{g}$ is integrable

28. Which of the following is not true for any refinement P* of a partition P of [a, b] and any bounded function f defined on [a, b] ?

(A) $L(P, f) \leq L(P^*, f)$

(B) $U(P, f) \leq U(P^*, f)$

(C) $L(P^*, f) \leq U(P^*, f)$

(D) $U(P^*, f) \leq U(P, f)$

29. What is the value of $\Gamma\left(\frac{5}{2}\right)$?

(A) $3\sqrt{\pi}$

(B) $3\frac{\sqrt{\pi}}{4}$

(C) $3\frac{\sqrt{\pi}}{2}$

(D) $\frac{\sqrt{\pi}}{2}$

30. The value of $\iiint_A e^{x+y+z} dxdydz$, where $A = \{(x, y, z), 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1\}$ is :

(A) e^2

(B) e^3

(C) $(e-1)^2$

(D) $(e-1)^3$

31. The value of $\iint_A \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$, where $A = \{(x, y), 0 \leq x \leq 1, 0 \leq y \leq 1\}$, is :

(A) $\frac{\pi^2}{4}$

(B) $\frac{\pi^2}{2}$

(C) $\frac{\pi^2}{3}$

(D) $\frac{\pi^2}{8}$

32. If $r = \sqrt{x^2+y^2+z^2}$, then $\nabla^2(r^n) =$

(A) nr^{n-2}

(B) $(n+1)r^{n-2}$

(C) $n(n+1)r^{n-1}$

(D) $n(n+1)r^{n-2}$

33. Which of the following is not true ?

(A) Every group is a subgroup of itself

(B) Every group has exactly two improper subgroups

(C) Every cyclic group is abelian

(D) Every group of prime order is simple

34. Which of the following functions from R to R is a permutation of R ?

(A) $f(x) = x + 1$

(B) $g(x) = x^2$

(C) $h(x) = e^x$

(D) None of these

◊◊

35. Which of the following is not a homomorphism ?

(A) $\phi : F \to R$ defined by $\phi(f) = 3f$, $f \in F$, where F is the additive group of all functions mapping R into R and R is the additive group of all real numbers

(B) $\phi : M_n \to R$ defined by $\phi(A) = Tr(A), A \in M_n$, where M_n is the additive group of all $n \times n$ matrices with real entries and R is the additive group of all real numbers

(C) $\phi : F \to R^*$ defined by $\phi(f) = \int_0^1 f(x)dx, f \in F$, where F is the multiplicative group of all continuous functions from R into R that are non-zero at every $x \in R$ and R^* is the multiplicative group of all non-zero real numbers

(D) $\phi : R^* \to R^*$ defined by $\phi(x) = |x|$, $x \in R^*$, where R^* is the multiplicative group of all non-zero real numbers

36. Which of the following is a sufficient condition for a ring R to be commutative ?

(A) $a^2 = a, \forall\, a \in R$

(B) $a^3 = a, \forall\, a \in R$

(C) Both (A) and (B)

(D) Neither (A) nor (B)

37. Which of the following is/are true in a vector space V over the field F ?

(A) $ax = 0_V \Rightarrow a = 0_F$ or $x = 0_V$

(B) $a.0_V = 0_V$

(C) $0_F.x = 0_V$

(D) All of these

38. If P and Q are subspaces of a vector space V_F, which of the following is not a subspace of V_F ?

(A) $P \cap Q$

(B) $P \cup Q$

(C) $P + Q$

(D) $aP + bQ, a, b \in F$

39. The dimension of the vector space C of complex numbers over the field R of real numbers is :

(A) 1

(B) 2

(C) 3

(D) None of these

40. If V_F is a finite-dimensional vector space of dimension n and k is the kernel of a homomorphism from V_F onto F_F, then the dimension of the quotient space $\frac{V}{K}$ is equal to :

(A) 0

(B) 1

(C) n

(D) None of these

41. If $f(x) \to 0$ as $x \to 0$, then $\lim_{x \to 0} \frac{e^{f(x)} - 1}{f(x)} =$

(A) 0

(B) 1

(C) e

(D) –1

42. Which of the following is a point of discontinuity of the second kind of the function

$f(x) = 1, x = 0$

$= x + \frac{1}{2}, 0 < x < \frac{1}{2}$

$= \frac{1}{2}, x = \frac{1}{2}$

$= x + \frac{1}{2}, \frac{1}{2} \leq x < 1$

$= 0, x = 1$?

(A) $x = 0$

(B) $x = \frac{1}{2}$

(C) $x = 1$

(D) None of these

43. The coefficient of $\left(x - \frac{\pi}{2}\right)^4$, when sin x is expanded as a Taylor series in $\left(x - \frac{\pi}{2}\right)$, is :

(A) 1

(B) $-\frac{1}{2}$

(C) $\frac{1}{24}$

(D) $\frac{1}{720}$

44. If $2y = x(1 + y_1)$, then $y_3 =$

(A) $x^2 + y^2$

(B) $x^2 - y^2$

(C) 0

(D) $x + y$

45. The angle of intersection of the curves $r = a \cos \theta$ and $r = a(1 - \cos \theta)$ is :

(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{3\pi}{2}$

(D) $\frac{5\pi}{6}$

46. Which of the following is true for the function $u = \sin^{-1}\left(\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}\right)$?

(A) $\frac{\partial u}{\partial x} = -\frac{\partial u}{\partial y}$

(B) $\frac{\partial u}{\partial x} = -\frac{x}{y}\frac{\partial u}{\partial y}$

(C) $\frac{\partial u}{\partial x} = -\frac{y}{x}\frac{\partial u}{\partial y}$

(D) $\frac{\partial u}{\partial x} = -xy\frac{\partial u}{\partial y}$

47. The maximum value of the radius of curvature of the ellipse

$p^2 = a^2 \cos^2 \psi + b^2 \sin^2 \psi$

is :

(A) ab

(B) $\frac{b^2}{a}$

(C) $\frac{a^2}{b}$

(D) $\frac{a}{b}$

◊◊

48. Which of the following is not an asymptote of the curve

$$x^3+2x^2y-xy^2-2y^3+4y^2+2xy+y-1=0\ ?$$

(A) $y=x+1$

(B) $y=-x+1$

(C) $y=-\frac{1}{2}x$

(D) $x+y=0$

49. The equation $|z+1|+|z-1|=4$ represents in the z-plane :

(A) A circle

(B) An ellipse

(C) A square

(D) A rectangle

50. If $x_r=\cos\frac{\pi}{2^r}+i\sin\frac{\pi}{2^r}$, $r=1, 2, 3,\ldots\ldots$, then

$\prod_{r=1}^{\infty}x_r=$

(A) 1

(B) $\frac{\pi}{2}$

(C) π

(D) -1

51. If $x+iy=\cosh(u+iv)$, then :

(A) $\frac{x^2}{\cosh^2 u}+\frac{y^2}{\sinh^2 u}=1$

(B) $\frac{x^2}{\cosh^2 u}-\frac{y^2}{\sinh^2 u}=1$

(C) $\frac{x^2}{\sinh^2 u}+\frac{y^2}{\cosh^2 u}=1$

(D) $\frac{x^2}{\sinh^2 u}-\frac{y^2}{\cosh^2 u}=1$

52. $\sin(\log i^i)=$

(A) 1

(B) 0

(C) -1

(D) i

53. If the point (α, β) lies outside the parabola $y^2=4ax$, then $\beta^2-4a\alpha$ is :

(A) Zero

(B) Positive

(C) Negative

(D) None of these

54. The eccentricity of the ellipse

$$4x^2+y^2-8x+2y+1=0$$

is :

(A) $\sqrt{3}$

(B) $\frac{\sqrt{3}}{2}$

(C) $2\sqrt{3}$

(D) $\frac{2}{\sqrt{3}}$

55. The line $lx+my+n=0$ will touch the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ if :

(A) $a^2l^2+b^2m^2=n^2$

(B) $al^2+bm^2=n^2$

(C) $l^2+m^2=n^2$

(D) $a^2l^2-b^2m^2=n^2$

56. The center of the conic

$$21x^2 - 6xy + 29y^2 + 6x - 58y - 151 = 0$$

is :

(A) (1, 0)
(B) (0, 0)
(C) (0, 1)
(D) (1, 1)

57. The radius of the circle in which the plane $x+2y-z = 4$ cuts the sphere $x^2+y^2+z^2-x+z-2 = 0$ is :

(A) $\frac{3}{\sqrt{6}}$

(B) $\sqrt{\frac{5}{2}}$

(C) 3

(D) 1

58. The semi-vertical angle of the right circular cone having its vertex at the origin and passing through the circle $x^2 + z^2 = 25$, $y = 4$ is :

(A) $\tan^{-1}\left(\frac{5}{4}\right)$

(B) $\tan^{-1}\left(\frac{4}{5}\right)$

(C) $\tan^{-1}(5)$

(D) $\tan^{-1}(4)$

59. The locus of the tangent lines to a sphere which are parallel to a given line is :

(A) A sphere
(B) A cone
(C) A cylinder
(D) None of these

60. The surface represented by $\frac{x^2}{2} - \frac{y^2}{3} = z$ is :

(A) An elliptic paraboloid
(B) A hyperbolic paraboloid
(C) A hyperboloid of one sheet
(D) A hyperboloid of two sheets

Sr. No. ..... 0095 .....

ENTRANCE TEST-2017

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

MATHEMATICS

Question Booklet Series **A**

Total Questions : **60**

Time Allowed : **70 Minutes**

Roll No. : | | | | | | |

Instructions for Candidates :

1. Write your Roll Number in the space provided at the top of this page of Question Booklet and fill up the necessary information in the spaces provided on the OMR Answer Sheet.
2. OMR Answer Sheet has an Original Copy and a Candidate's Copy glued beneath it at the top. While making entries in the Original Copy, candidate should ensure that the two copies are aligned properly so that the entries made in the Original Copy against each item are exactly copied in the Candidate's Copy.
3. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
4. Choose the correct / most appropriate response for each question among the options A, B, C and D and darken the circle of the appropriate response completely. The incomplete darkened circle is not correctly read by the OMR Scanner and no complaint to this effect shall be entertained.
5. Use only blue/black ball point pen to darken the circle of correct/most appropriate response. In no case gel/ink pen or pencil should be used.
6. Do not darken more than one circle of options for any question. A question with more than one darkened response shall be considered wrong.
7. There will be 'Negative Marking' for wrong answers. Each wrong answer will lead to the deduction of 0.25 marks from the total score of the candidate.
8. Only those candidates who would obtain positive score in Entrance Test Examination shall be eligible for admission.
9. Do not make any stray mark on the OMR sheet.
10. Calculators and mobiles shall not be permitted inside the examination hall.
11. Rough work, if any, should be done on the blank sheets provided with the question booklet.
12. OMR Answer sheet must be handled carefully and it should not be folded or mutilated in which case it will not be evaluated.
13. Ensure that your OMR Answer Sheet has been signed by the Invigilator and the candidate himself/herself.
14. At the end of the examination, hand over the OMR Answer Sheet to the invigilator who will first tear off the original OMR sheet in presence of the Candidate and hand over the Candidate's Copy to the candidate.

1. Which of the following are the points of discontinuity of the function f(x) in the interval [0, 1] where

$f(x) = 2$, when $x = 0$

$= 4x^2 + 1,\ 0 < x < ½$

$= 2x + 1,\ ½ \le x \le 1$?

(A) $x = 0$ and $x = ½$ only

(B) $x = ½$ only

(C) $x = 0$ only

(D) None of the above

2. What is the coefficient of x^2 in the power series expansion of $\log(1 + e^x)$?

(A) $\frac{1}{8}$

(B) $\frac{1}{4}$

(C) 2

(D) None of the above

3. Which of the following is the kth derivative of the function $y = \text{Sin}x\text{Cos}x$?

(A) $y_k = 2^k \text{Sin}(x + k\pi)$

(B) $y_k = 2^k \text{Cos}(x + k\pi)$

(C) $y_k = 2^{k-1} \text{Cos}\left(2x + \frac{k\pi}{2}\right)$

(D) $y_k = 2^{k-1} \text{Sin}\left(2x + \frac{k\pi}{2}\right)$

4. What is value of $\lim_{x \to 0} \frac{\text{Sin}\, x - x + 2x^3}{x^3}$?

(A) 2

(B) $\frac{13}{6}$

(C) $\frac{5}{3}$

(D) $\frac{11}{6}$

5. If S is the length of an arc of the curve $f(r, Q) = 0$, then which of the following is true ?

(A) $\left(\frac{dS}{dQ}\right)^2 = 1 + \left(\frac{dQ}{dr}\right)^2$

(B) $\left(\frac{dS}{dQ}\right)^2 = 1 + r^2\left(\frac{dr}{dQ}\right)^2$

(C) $\left(\frac{dS}{dQ}\right)^2 = r^2\left(\frac{dr}{dQ}\right)^2$

(D) None of the above

6. If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$,

then for which of the following value of f(u),

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = f(u) \ ?$$

(A) sin u

(B) tan u

(C) ½ sin 2u

(D) ½ tan 2u

7. How many asymptotes does the curve $y^2 = 0x$ have ?

(A) None

(B) One

(C) Two

(D) More than two

8. How many double points are on the curve ?

$x^4 + y^3 + 2x^3 + 3y^2 = 0$?

(A) Four

(B) Three

(C) Two

(D) One

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9. Let Z_1 and Z_2 be two complex numbers, then

$$|Z_1 + Z_2|^2 = |Z_1|^2 + |Z_2|^2$$

if and only if

(A) $Z_1 - Z_2$ is purely imaginary

(B) $\frac{Z_1}{Z_2}$ is purely imaginary

(C) $\frac{Z_1}{Z_2}$ is purely real

(D) None of the above

10. Which of the following is the value of $(1+\omega)^3$ where :

$$\omega = \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) ?$$

(A) $-i$

(B) i

(C) -1

(D) 1

11. Which of the following is the real part of $\cos^{-1}(i)$?

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) π

(D) None of the above

12. What is the sum to n terms of the series

$$\cos\theta + \cos 3\theta + \cos 5\theta + \ldots\ldots ?$$

(A) $\frac{\cos 2n\theta}{2\sin\theta}$

(B) $\frac{\sin 2n\theta}{2\sin\theta}$

(C) $\frac{\cos n\theta}{\sin\theta}$

(D) $\frac{\sin n\theta}{\cos\theta}$

13. For what value of k, the line $y = 3x + k$ touches the parabola $y^2 = 2x$?

(A) $\frac{2}{3}$

(B) $\frac{3}{2}$

(C) $\frac{5}{6}$

(D) None of the above

14. If CP is an equi-conjugate semi-diameter of an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

then which of the following is the length of CP ?

(A) $\frac{a+b}{2}$

(B) $\frac{ab}{2}$

(C) $\sqrt{\frac{a^2+b^2}{2}}$

(D) $\sqrt{a^2+b^2}$

15. If a circle and the rectangular hyperbola $xy = c^2$ mee in the four points k_1, k_2, k_3, k_4, then what is the valu of the product $k_1k_2k_3k_4$?

(A) c^4

(B) 1

(C) 2

(D) None of the above

16. Which of the following is the centre of the conic

$$x^2 - 5xy + y^2 + 8x - 12y + 15 = 0 ?$$

(A) $(-4, 0)$

(B) $(0, 4)$

(C) $\left(-4, -\frac{16}{3}\right)$

(D) None of the above

17. For what value of the constant k, the two spheres
$x^2 + y^2 + z^2 + 6y + 2z + k = 0$ and
$x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$?
(A) -6
(B) 4
(C) 8
(D) None of the above

18. What is the general equation of the cone of second degree which possess through the axes ?
(A) $ax^2 + by^2 + cz^2 = 0$
(B) $fyz + gzx + hxy = 0$
(C) $ax^2 + by^2 + cz^2 = xy + yz + zx$
(D) None of the above

19. Which of the following is the equation of cylinder which intersects the curve
$x^2 + y^2 + z^2 = 1$, $ux + vy + wz = d$
and whose generator are parallel to the axis of x ?
(A) $ux^2 + vy^2 + wz^2 = 1$
(B) $(u + v)x^2 + (v + w)y^2 + (w + u)z^2 = d$
(C) $u^2xy + v^2yz + w^2zx = d$
(D) None of the above

20. Tangent planes are drawn to the conicoid $ax^2 + by^2 + cz^2 = 1$ through the point (α, β, γ), then the perpendiculars to them from origin generate the conc. :
(A) $(\alpha x + \beta y + \gamma z)^2 = \left(\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c}\right)$
(B) $(axy + byz + czx)^2 = 1$
(C) $(ax + by + cz)^2 = (\alpha xy + \beta yz + \gamma zx)^2$
(D) None of the above

21. For which of the following values of $\psi(x)$,
$\int \tan^5 x\,dx - \frac{\tan^4 x}{4} = \psi(x)$?
(A) $\log \sec x + \tan^2 x$
(B) $\log \sec x - \frac{1}{2}\tan^2 x$
(C) $x - \tan x$
(D) $\tan^2 x - \log \sec x$

22. Which of the following is the value of
$\frac{1}{(D-2)^2}(x^3)$?
(A) $x^3 - 3x^2 + 9x - 3$
(B) $\frac{1}{8}(2x^3 + 3x^2 - 9x + 8)$
(C) $\frac{1}{8}(2x^3 + 6x + 9x + 6)$
(D) None of the above

23. Which of the following is the solution of differential equation
$y = 2px + y^2p^3$
(A) $y^2 = cx + \frac{c^3}{8}$
(B) $y = 2cx + c^3y^2$
(C) $y = 2cx^2 + c^3$
(D) None of the above

24. Let $P_n(x)$ denote the Legendre polynomials for $n = 0, 1, 2, \ldots$, then for which of following values of the pair (k, ℓ)
$(2n + 1)\,xP_n(x) = kP_{n+1}(x) + \ell P_{n-1}(x)$?
(A) $(2n - 1, 2n + 1)$
(B) $(n - 1, n + 1)$
(C) $(n, n - 1)$
(D) $(n + 1, n)$

25. For what value of the pair (m, n), the matrix
$$2m\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & n & -1 \end{bmatrix}$$
is the inverse of the matrix
$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$?
(A) (3, 1)
(B) (1, 3)
(C) $(-1, 3)$
(D) $(-1, -3)$

26. Let $A = \begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}$,

then for what value of the integer k, $A^{10} = 3^k A$?

(A) 9

(B) 6

(C) 3

(D) None of the above

27. What is the rank of the matrix

$$\begin{bmatrix} 1 & 3 & 2 & 1 \\ 3 & 1 & 4 & 2 \\ 5 & 7 & 8 & 4 \\ 7 & 5 & 10 & 5 \end{bmatrix}$$

(A) One

(B) Two

(C) Three

(D) Four

28. For which value of P, the vectors

$$\begin{bmatrix} 7 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 8 \\ P \\ 3 \end{bmatrix}$$

are linearly dependent ?

(A) 8

(B) 7

(C) – 6

(D) 5

29. Let $A = [a_{ij}]_{5 \times 4}$ be a matrix of order 5 × 4, then the columns of A are linearly independent if and only if :

(A) R(A) = 4

(B) R(A) = 5

(C) R(A) < 4

(D) None of the above

30. How many linearly independent solutions does the of equations

$x + y - 4z - 2u = 0$, $2x - 3y + 2z + 2u = 0$

have ?

(A) 4

(B) 3

(C) 2

(D) None

31. Let A be a matrix of order 3 × 5 with rank of A Then how many linearly independent solutions the system of non-linear homogeneous equ $AX = H$, $H \neq 0$, have ?

(A) 4

(B) 3

(C) 2

(D) None of the above

32. For which value of the pair (α, β), the matrix

$$\begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & \alpha \\ -\frac{1}{\sqrt{3}} & \beta & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \end{bmatrix}$$

is an orthogonal matrix ?

(A) $\left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{2}}\right)$

(B) $\left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{3}}\right)$

(C) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}\right)$

(D) None of the above

33. If α, β, γ are the roots of the equation

$$x^3 + 9x^2 + 23x + 15 = 0,$$

then which of the following is the value of

$$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} ?$$

(A) 5

(B) $\frac{23}{15}$

(C) $\frac{1}{4}$

(D) $\frac{1}{5}$

34. The roots of the equation

$$28x^3 + 39x^2 + 12x - 1 = 0$$

are in Harmonic progression. Which of the following is its mean root ?

(A) 4

(B) $\frac{1}{4}$

(C) $\frac{1}{7}$

(D) None of the above

35. Which of the following is the equation whose roots are less than 2 than that of the equation

$$x^3 + 4x^2 + 7x + 8 = 0 ?$$

(A) $y^3 + 10y^2 - 35y + 36 = 0$

(B) $y^3 - 10y^2 + 35y - 30 = 0$

(C) $y^3 + 6y^2 + 3y + 30 = 0$

(D) $y^3 + 10y^2 + 35y + 46 = 0$

36. Which of the following is the equation whose roots are the squares of the roots of the equation

$$x^3 - 2x^2 - x + 2 = 0 ?$$

(A) $y^3 + 6y^2 + 9y + 4 = 0$

(B) $y^3 + 9y^2 - 6y + 4 = 0$

(C) $y^3 - 6y^2 + 9y - 4 = 0$

(D) None of the above

37. What is the value of $S_3 = \alpha^3 + \beta^3 + \gamma^3$ where α, β, γ are the roots of the equation

$$x^3 - x^2 + 1 = 0 ?$$

(A) 4

(B) 3

(C) -2

(D) None of the above

38. The cubic equation

$$x^3 + 3x + 14 = 0$$

has :

(A) All the three roots real and distinct

(B) One real and two conjugate imaginary roots

(C) All the three roots not only real but two of them are also equal

(D) None of the above

39. The equation

$$x^4 - 5x^3 + 7x^2 - 5x + 6 = 0$$

has :

(A) no negative root

(B) at least one negative root

(C) at least two negative roots

(D) None of the above

40. How many roots does the polynomial equation :

$$4z^6 - 3z^5 + 3z^4 - 2z^3 + 2z^2 - 2z + 1 = 0$$

have within and on the circle $|z| \leq 1$?

(A) None

(B) Two

(C) Four

(D) Six

41. Which of the following statements are true ?

(A) The set of all integers is countable and set of all polynomial functions P_n with integer coefficients is countable

(B) The set of all integers is countable but set of all polynomial functions P_n with integer coefficients is not countable

(C) The set of all integers is not countable but set of all polynomial functions P_n with integer coefficients is countable

(D) None of the above

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42. How many limit points does the sequence $\{s_n\}$ where

$$S_n = (-1)^n \left\{\left(1 + \frac{1}{n}\right)\right\}, n \in N$$

have ?

(A) None
(B) One
(C) Two
(D) More than two

43. Consider the series $\sum \frac{1}{n^{3k}}$, then which of the following is true ?

(A) The series converges for $k < \frac{1}{3}$

(B) The series converges for $0 < k < \frac{1}{3}$

(C) The series diverges for $k > \frac{1}{3}$

(D) The series diverges for $k \leq \frac{1}{3}$

44. Let $f(x) = x^2$, $g(x) = \frac{1}{x}$ and $I = [-1, 1]$, then

(A) f(x) and g(x) are both uniformly continuous on I
(B) f(x) is uniformly continuous on I but g(x) is not uniformly continuous on I
(C) f(x) and g(x) are not both uniformly continuous on I
(D) f(x) is not uniformly continuous on I but g(x) is uniformly continuous on I

45. Consider the function

$$f(x) = \begin{cases} 0 & \text{when x is rational} \\ 1 & \text{when x is irrational} \end{cases}$$

over the interval $[-1, 2]$. If U(P, f) is the upper Riemann sum of f relative to a partition P, then U(P, f) =

(A) 2
(B) 3
(C) 0
(D) None of these

46. Consider the two functions f(x) and g(x) defined by

$$f(x) = \begin{cases} 3 & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

$$g(x) = \begin{cases} 0 & \text{when x irrational or zero} \\ \frac{1}{q} & \text{when } x = p/q \text{ is a non-zero rational} \\ & \text{number and } \frac{p}{q} \text{ in lowest form} \end{cases}$$

over the interval $I = [0, 1]$. Then which of the following is true ?

(A) f(x) and g(x) are both integrable on I
(B) f(x) is integrable on I but g(x) is not integrable on I
(C) f(x) and g(x) both are not integrable on I
(D) None of the above

47. What is the value of integral

$$\int_0^4 [x]dx \ ?$$

(A) 8
(B) 4
(C) 6
(D) None of the above

48. If a function f(x) is continuous on [2, 5], then there exists a point 'c' in [2, 5] such that

$$\int_2^5 f(x)dx =$$

(A) 3c
(B) cf(2)
(C) cf(5)
(D) 3f(c)

49. For what value of k, the function f(x, y) defined by

$$f(x, y) = \begin{cases} \dfrac{xy^k}{x^2 + y^{2k}} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

is continuous at (0, 0) ?

(A) $k = 1$

(B) $k = 2$

(C) $k = 3$

(D) None of these

50. If $xyz = t_1t_2t_3$, what is the minimum value of $t_2t_3x + t_3t_1y + t_1t_2z$?

(A) $t_1 + t_2 + t_3$

(B) $3t_1t_2t_3$

(C) $t_1t_2t_3$

(D) None of the above

51. If $\beta(m, n)$ denotes the beta function, which of the following is the value of $\beta\left(\frac{3}{2}, \frac{1}{2}\right)$?

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) None of the above

52. What is the value of the double integral

$$\iint_R (x^2 + y^2)dxdy$$

where R is the region defined by $R = \{(x, y) : 0 < x < y < 1\}$?

(A) 1

(B) $\frac{2}{3}$

1

above

53. Let G be a group of all 2×2 non-singular matrices over the real numbers, then which of the following represent a member Z(G), the centre of G ?

(A) $\begin{bmatrix} a & a \\ a & 0 \end{bmatrix}$

(B) $\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

(C) $\begin{bmatrix} 0 & a \\ a & 0 \end{bmatrix}$

(D) None of the above

54. Let G be a finite group with order O(G). If N is a normal sub-group of G, then which of the following is true ?

(A) $O\left(\frac{G}{N}\right) = O(G) - O(N)$

(B) $O\left(\frac{G}{N}\right) = O(G).O(N)$

(C) $O\left(\frac{G}{N}\right) = \frac{O(G)}{O(N)}$

(D) $O\left(\frac{G}{N}\right) = O(G) + O(N)$

55. If a = (1 2 3 5) and b = (1 5 7 9), then which of the following is equal to $a^{-1}ba$? Where $(f \circ g)(x) = f(g(x))$

(A) (3 7 9 5)

(B) (5 7 9 3)

(C) (1 2 7 8 9)

(D) None of the above

DAJ-n over

56. For which of the following value of k,

$$Z_k = \{0, 1, 2, ..., k-1\},$$

the ring of integers mod k, is a field ?

(A) 256

(B) 961

(C) 1147

(D) None of the above

57. Let operation of addition and scalar multiplication in

$V = \{(a, b) : a, b \in R\}$

be defined as follows :

(I) $(a, b) + (c, d) = (0, b + d)$ and

$k(a, b) = (0, kb)$

(II) $(a, b) + (c, d) = (a + c, b + d)$ and

$k(a, b) = (0, kb)$.

Then which of the following is true ?

(A) V is a vector space over R with respect to operation defined by (I)

(B) V is the vector space over R with respect to operation defined by (II)

(C) V is not a vector space with respect to operations defined by (I) and (II)

(D) None of the above

58. If W_1 and W_2 are two sub spaces of a finite dimensional vector space V, then $\dim(W_1 \cap W_2) =$

(A) $\dim(W_1) + \dim(W_2)$

(B) $\dim(W_1) + \dim(W_2) - \dim(W_1 + W_2)$

(C) $\dim(W_1) \dim(W_2)$

(D) $\dim(W_1) / \dim(W_2)$

59. If S and T are non-empty subsets of a vector space V and L(S) is the linear span of S, then which of the following is true ?

(A) L(S) is a sub space of V and $L(S) \subset L(T)$ if $S \subset T$

(B) L(S) is a sub space of V and $L(S) \supset L(T)$ if $S \subset T$

(C) L(S) is a not a sub space of V

(D) None of the above

60. Given two vector spaces V and W over field F, let Hom(V, W) be the set of all vector space homomorphisms of V into W. If dim(V) = 19 and dim(W) = 21, then what is the dim Hom(V, W) ?

(A) 40

(B) 19

(C) 21

(D) None of the above

Sr. No. 1545

ENTRANCE TEST-2016

FACULTY OF PHYSICAL & MATERIAL SCIENCES

M.A./M.Sc. MATHEMATICS

Total Questions : 60
Time Allowed : 70 Minutes

Question Booklet Series A

Roll No. : | | | | | | |

Instructions for Candidates :

1. Write your Roll Number in the space provided at the top of this page of Question Booklet and fill up the necessary information in the spaces provided on the OMR Answer Sheet.
2. OMR Answer Sheet has an Original Copy and a Candidate's Copy glued beneath it at the top. While making entries in the Original Copy, candidate should ensure that the two copies are aligned properly so that the entries made in the Original Copy against each item are exactly copied in the Candidate's Copy.
3. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
4. Choose the correct / most appropriate response for each question among the options A, B, C and D and darken the circle of the appropriate response completely. The incomplete darkened circle is not correctly read by the OMR Scanner and no complaint to this effect shall be entertained.
5. Use only blue/black ball point pen to darken the circle of correct/most appropriate response. In no case gel/ink pen or pencil should be used.
6. Do not darken more than one circle of options for any question. A question with more than one darkened response shall be considered wrong.
7. There will be 'Negative Marking' for wrong answers. Each wrong answer will lead to the deduction of 0.25 marks from the total score of the candidate.
8. Only those candidates who would obtain positive score in Entrance Test Examination shall be eligible for admission.
9. Do not make any stray mark on the OMR sheet.
10. Calculators and mobiles shall not be permitted inside the examination hall.
11. Rough work, if any, should be done on the blank sheets provided with the question booklet.
12. Ensure that your OMR Answer Sheet has been signed by the Invigilator and the candidate himself/herself.
13. OMR Answer sheet must be handled carefully and it should not be folded or mutilated in which case it will not be evaluated.
14. At the end of the examination, hand over the OMR Answer Sheet to the invigilator who will first tear off the original OMR sheet in presence of the Candidate and hand over the Candidate's Copy to the candidate.

SEAL

1. Which of the following is true for the function $f(x) = |x|$?

   (A) It is continuous for all x

   (B) It is differentiable for all x

   (C) It is continuous as well as differentiable for all x

   (D) It is neither continuous nor differentiable at $x = 0$

2. If $f(x) = e^{\frac{1}{x}}, x \neq 0$
   $= 0, x = 0$

   then $\lim_{x \to 0} f(x)$:

   (A) $= 0$ (B) $= 1$

   (C) $= \infty$ (D) Does not exist

3. At $x = 0$, the function :

$$f(x) = \frac{\sin 2x}{x}, x \neq 0$$
$$= 1, x = 0$$

   is:

   (A) Continuous

   (B) Differentiable

   (C) Continuous but not differentiable

   (D) Neither continuous nor differentiable

4. If $f'(x)$ is positive in $[a, b]$, then :

   (A) $f(a) < f(b)$ (B) $f(a) > f(b)$

   (C) $f(a) = f(b)$ (D) None of the above holds

5. The coefficient of $\left(x - \frac{\pi}{4}\right)^3$ when sin x is expanded in ascending powers of $\left(x - \frac{\pi}{4}\right)$ is :

(A) $-\frac{1}{\sqrt{2}}$ (B) $-\frac{1}{2\sqrt{2}}$

(C) $-\frac{1}{6\sqrt{2}}$ (D) $-\frac{1}{4\sqrt{2}}$

6. $\lim_{x \to 0} (1 - x)^{\frac{1}{x}} =$

(A) 1 (B) $\frac{1}{e}$

(C) e (D) ∞

7. The points of minimum curvature on the curve $y = \log \sin x$ are, for any integer n, given by x =

(A) $\pm\frac{n\pi}{2}$ (B) $\pm\frac{n\pi}{3}$

(C) $\pm n\pi$ (D) $\pm\frac{n\pi}{4}$

8. Which of the following curves has no asymptotes ?

(A) $y^2 = 4ax$ (B) $x^2y^2 + y^2 = 1$

(C) $xy^3 + x^3y = 1$ (D) $xy^2 + y^2 = x$

9. The general value of x which satisfies the equation :

$\text{cis } x. \text{ cis } 2x. \text{ cis } 3x \ldots\ldots \text{ cis } nx = 1$

is :

(A) $\frac{2m\pi}{n+1}$ (B) $\frac{2m\pi}{n(n+1)}$

(C) $\frac{4m\pi}{n(n+1)}$ (D) $\frac{4m\pi}{n}$

where m is an integer.

10. If $\cos\theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$, $\cos\phi = \frac{1}{2}\left(y + \frac{1}{y}\right)$, then one of the values of

$x^m y^n + \frac{1}{x^m y^n}$ is :

(A) $2\cos(\theta + \phi)$ (B) $2\cos(m\theta + n\phi)$

(C) $2\cos(n\theta + m\phi)$ (D) $2\cos(m\theta - n\phi)$

11. $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} =$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ (D) π

12. The sum to n terms of the series

$\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + ...$

is equal to :

(A) $\tan^{-1}\frac{n}{n+2}$ (B) $\tan^{-1}\frac{n+2}{n}$

(C) $\tan^{-1}\frac{1}{n+2}$ (D) $\tan^{-1}\frac{1}{n+1}$

13. The line $y = mx + \frac{a}{m}$ touches the parabola $y^2 = 4ax$ at the point :

(A) $(am^2, 2am)$ (B) $\left(am^2, \frac{2a}{m}\right)$

(C) $\left(\frac{a}{m^2}, 2am\right)$ (D) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

14. The distance between the foci of the ellipse $3x^2 + 4y^2 = 12$ is equal to :

(A) 4 (B) 3

(C) 2 (D) 1

15. If a circle cuts the rectangular hyperbola $xy = a^2$ in points t_1, t_2, t_3, t_4, then $t_1t_2t_3t_4 =$

(A) 1 (B) 2

(C) a^2 (D) a^4

16. The conditions for the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ to lie in the plane $ax + by + cz + d = 0$ are :

(A) $a\alpha + b\beta + c\gamma = 0, al + bm + cn + d = 0$

(B) $a\alpha + b\beta + c\gamma = 0, al + bm + cn = 0$

(C) $a\alpha + b\beta + c\gamma + d = 0, al + bm + cn = 0$

(D) $a\alpha + b\beta + c\gamma + d = 0, al + bm + cn + d = 0$

17. The equation of the radical plane of two spheres

$x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$

and

$x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$

is :

(A) $(u_1 - u_2)x + (v_1 - v_2)y + (w_1 - w_2)z + (d_1 - d_2) = 0$

(B) $(u_1 + u_2)x + (v_1 + v_2)y + (w_1 + w_2)z + (d_1 + d_2) = 0$

(C) $2(u_1 - u_2)x + 2(v_1 - v_2)y + 2(w_1 - w_2)z + (d_1 - d_2) = 0$

(D) $2(u_1 + u_2)x + 2(v_1 + v_2)y + 2(w_1 + w_2)z + (d_1 + d_2) = 0$

18. The general equation to the cone of the second degree passing through the axes is :

(A) $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$

(B) $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$

(C) $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$

(D) $fyz + gzx + hxy = 0$

19. The surface represented by $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$ is :

(A) An ellipsoid

(B) A hyperboloid of one sheet

(C) A sphere

(D) A cone

20. The condition for the plane $\lambda x + my + nz = p$ to touch the central conicoid $ax^2 + by^2 + cz^2 = 1$ is :

(A) $a^2l^2 + b^2m^2 + c^2n^2 = p^2$

(B) $al^2 + bm^2 + cn^2 = p^2$

(C) $\frac{l^2}{a} + \frac{m^2}{b} + \frac{n^2}{c} = p^2$

(D) $\frac{l^2}{a^2} + \frac{m^2}{b^2} + \frac{n^2}{c^2} = p^2$

21. $\int \frac{dx}{x^2\sqrt{1+x^2}} =$

(A) $\frac{x}{\sqrt{1+x^2}}$

(B) $-\frac{x}{\sqrt{1+x^2}}$

(C) $\frac{\sqrt{1+x^2}}{x}$

(D) $-\frac{\sqrt{1+x^2}}{x}$

22. The value of $\int_0^{\pi} \frac{\sin nx}{\sin x} dx$, where n is an odd integer, is :

(A) 0

(B) π

(C) n

(D) $n\pi$

23. The area bounded by the curve $y = \cos x (0 \le x \le 2\pi)$, the x-axis and the ordinate $x = 2\pi$ is equal to :

(A) 2

(B) 2π

(C) 4π

(D) 4

24. For what value of n is $\text{div}\bar{F} = 0$, where $\bar{F} = r^n\vec{r}, \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$?

(A) –2 (B) –1

(C) –3 (D) – 4

25. The degree and order of the differential equation

$$x\frac{dy}{dx} + y = a\frac{dx}{dy}$$

are respectively :

(A) 2, 1 (B) 1, 2

(C) 1, 1 (D) 2, 2

26. The solution of the differential equation

$x(1 + y^2)dx + y(1 + x^2)dy = 0$

is :

(A) $1 + x^2 + y^2 = k$ (B) $(1 + x^2)(1 + y^2) = k$

(C) $\frac{1+x^2}{1+y^2} = k$ (D) $\frac{1+y^2}{1+x^2} = k$

27. Which of the following differential equations is not exact ?

(A) $xdy + (y - x)dx = 0$ (B) $(e^y + 1)\cos x dx + e^y \sin x dy = 0$

(C) $(x^2 + y^2)dx + (x^2 - y^2)dy = 0$ (D) $(1 - 2xy - y^2)dx - (x + y)^2 dy = 0$

28. The particular integral of the differential equation

$(D^3 - 3D + 2)y = x^2e^x$

is :

(A) $e^x\left(\frac{x^4}{36} - \frac{x^3}{27} + \frac{x^2}{27}\right)$ (B) $e^x\left(\frac{x^4}{27} - \frac{x^3}{27} + \frac{x^2}{36}\right)$

(C) $e^x\left(\frac{x^4}{27} - \frac{x^3}{36} + \frac{x^2}{27}\right)$ (D) $e^x\left(\frac{x^4}{36} + \frac{x^3}{27} + \frac{x^2}{27}\right)$

29. If A is a square matrix and $P = A + \overline{A}'$, $Q = A - \overline{A}'$ then :

(A) P is symmetric, Q is skew-symmetric

(B) P is skew-symmetric, Q is symmetric

(C) P is Hermitian, Q is skew-Hermitian

(D) P is skew-Hermitian, Q is Hermitian

30. Which of the following is not true for any three square matrices P, Q, R of the same order ?

(A) $P + (Q + R) = (P + Q) + R$ (B) $P(QR) = (PQ)R$

(C) $P(Q + R) = PQ + PR$ (D) $PQ = PR \Rightarrow Q = R$

31. For what values of α, β, γ, the matrix

$$A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$$

satisfies $AA' = I$?

(A) $\alpha = \pm\frac{1}{\sqrt{6}}, \beta = \pm\frac{1}{\sqrt{2}}, \gamma = \pm\frac{1}{\sqrt{3}}$

(B) $\alpha = \pm\frac{1}{\sqrt{3}}, \beta = \pm\frac{1}{\sqrt{2}}, \gamma = \pm\frac{1}{\sqrt{6}}$

(C) $\alpha = \pm\frac{1}{\sqrt{2}}, \beta = \pm\frac{1}{\sqrt{3}}, \gamma = \pm\frac{1}{\sqrt{6}}$

(D) $\alpha = \pm\frac{1}{\sqrt{2}}, \beta = \pm\frac{1}{\sqrt{6}}, \gamma = \pm\frac{1}{\sqrt{3}}$

32. If A is an idempotent matrix of order 3 whose trace is equal to 5, then the trace of $5A^4 - 4A^3 + 3A^2 - 4A + I$ is equal to :

(A) 5 (B) 4

(C) 3 (D) 2

33. The sum and the product of the roots of the characteristic equation of the matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

are respectively :

(A) 7 and 7 (B) 7 and 5

(C) 7 and 6 (D) 7 and 8

34. For what value of b does the system of equations $2x + y + 2z = 0$, $x + y + 3z = 0$, $4x + 3y + bz = 0$ have a non-trivial solution ?

(A) $b = 2$ (B) $b = 3$

(C) $b = 6$ (D) $b = 8$

35. The rank of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix}$ is equal to :

(A) 2 (B) 3

(C) 1 (D) 4

36. The number of linearly independent solutions of the equation $x + y = 1$ is :

(A) 1 (B) 2

(C) 3 (D) 4

37. The condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ are in G.P. is that :

(A) $q^3 = p^3r$ (B) $p^3 = q^3r$

(C) $r^3 = p^3q$ (D) $pqr = 1$

38. If $P(x) = 0$ is an equation of odd degree such that $x^n P\left(\frac{1}{x}\right) = P(x)$, which of the following is a root of $P(x) = 0$?

(A) $x = 1$ (B) $x = -1$

(C) $x = 0$ (D) None of these

39. If α, β, γ are the roots of the cubic $x^3 + px^2 + qx + r = 0$, then $\alpha^3\beta^3 + \beta^3\gamma^3 + \gamma^3\alpha^3 =$

(A) $3r - pq$ (B) $\frac{q^2 - 2pr}{r^2}$

(C) $2(p^2 - 3q)$ (D) $3r^2 + q^3 - 3pqr$

40. How many roots of the equation $z^4 + z^2 + 1 = 0$ lie in $|z| < 1$?

(A) 1 (B) 2

(C) 3 (D) None

41. If the sequences $\{a_n\}$ and $\{b_n\}$ are defined by

$a_n = 2^n$, when n is odd

$= 3^n$, when n is even

and

$b_n = 3^n$, when n is odd

$= 2^n$, when n is even,

then the limit superior of the sequence $\left\{(a_n b_n)^{\frac{1}{n}}\right\}$ is equal to :

(A) 2 (B) 3

(C) 6 (D) 9

42. The series $\frac{x}{1} + \frac{1}{2}\cdot\frac{x^3}{3} + \frac{1.3}{2.4}\cdot\frac{x^5}{5} + \frac{1.3.5}{2.4.6}\cdot\frac{x^7}{7} + \ldots\ldots$ is convergent if :

(A) $x^2 > 1$ (B) $x^2 \leq 1$

(C) $x > 1$ (D) $x < 1$

43. Under what condition does the equation $ax = e^x$ have a root between 0 and 1 ?

(A) $a = e$ (B) $a < e$

(C) $a > e$ (D) None

44. Under what condition is $|x + y| < |x| + |y|$; $x, y \in R$?

(A) $xy > 0$ (B) $xy = 0$

(C) $xy < 0$ (D) $xy \leq 0$

45. Which of the following is true for the function f defined over [a, b] by

$f(x) = x$, when x is rational

$= -x$, when x is irrational ?

(A) f is integrable but $|f|$ is not integrable over [a, b]

(B) $|f|$ is integrable but f is not integrable over [a, b]

(C) f and $|f|$ are both integrable over [a, b]

(D) Neither f nor $|f|$ is integrable over [a, b]

46. Which of the following is not true ?

(A) Every continuous function on [a, b] is integrable over [a, b]

(B) Every monotonic function on [a, b] is integrable over [a, b]

(C) Every integrable function over [a, b] is continuous on [a, b]

(D) A bounded function having a finite number of discontinuities on [a, b] is integrable over [a, b]

47. If f is a bounded function defined on [a, b] and P_1, P_2 are any two partitions of [a, b], then :

(A) $L(P_1 \cup P_2, f) \leq L(P_1, f)$ (B) $U(P_1 \cup P_2, f) \geq U(P_1, f)$

(C) $L(P_1 \cup P_2, f) \leq U(P_1 \cup P_2, f)$ (D) $U(P_1, f) \leq U(P_2, f)$

48. The value of the R-integral $\int_{-1}^{1} f(x)dx$, where $f(x) = |x|$ is :

(A) 0 (B) 1

(C) 2 (D) 3

49. $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2 + y^6}$

(A) $= 0$ (B) $= \infty$

(C) $= 1$ (D) Does not exist

50. At (0, 0) the function

$$f(x,y) = \frac{x^2y}{x^3 + y^3}, (x,y) \neq (0,0)$$
$$= 0, (x, y) = (0, 0)$$

is :

(A) Continuous (B) Differentiable

(C) Discontinuous (D) Continuous but not differentiable

51. The value of $\iint_A xyf(x,y)dxdy$, where $A = \{(x, y), 0 < x < 1, 0 < y < 1\}$ and $f(x, y) = x + y$ is :

(A) $\frac{7}{12}$ (B) $\frac{11}{144}$

(C) $-\frac{1}{11}$ (D) $\frac{1}{3}$

52. The function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ has a maximum value at :

(A) $(1, 2)$ (B) $(-1, 2)$

(C) $(1, -2)$ (D) $(-1, -2)$

53. Let the binary operation * be defined on the set Z of integers by $a*b = a+b-7$. Then the inverse of 2 in Z is :

(A) -2 (B) 0

(C) 10 (D) 12

54. A non-commutative group has :

(A) At least 4 elements (B) At least 5 elements

(C) At least 6 elements (D) None of these

55. The number of generators of an infinite cyclic group is :

(A) 0 (B) 1

(C) 2 (D) Infinite

56. If $f : G \to G'$ is an isomorphism, then the kernel of f is equal to :

(A) (e) (B) G

(C) G' (D) None of these

57. Which of the following is incorrect in the theory of rings ?

(A) Every ideal is a subring

(B) The product of any two ideals is again an ideal

(C) Every field is an integral domain

(D) None of the above

58. The ideal (2) in the ring of integers is :

(A) A prime ideal (B) A principal ideal

(C) A maximal ideal (D) All of the above

59. Which of the following is true for the vectors (1, 0, 0), (0, 1, 0) and (0, 0, 1) in the real vector space R^3 ?

(A) They are linearly dependent in R^3 (B) They form a basis of R^3

(C) Both (A) and (B) (D) Neither (A) nor (B)

60. If V and W are vector spaces over the same field F of dimensions m and n respectively, then Hom(V, W) is a vector space over F of dimension :

(A) m^2 (B) mn

(C) n^2 (D) $\frac{m}{n}$

ROUGH WORK

1. For what values of m, n, the system of equations
$x + y + z = 6, x + 2y + 3z = 10, x + 2y + mz = n$
have no solution ?
(A) $m = 3, n = 10$ (B) $m = 3, n \neq 10$
(C) $m \neq 3, n = 10$ (D) $m \neq 3, n \neq 10$

2. If α, β, γ are the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$, then the equation whose roots are $\alpha\beta, \beta\gamma, \gamma\alpha$ is :
(A) $x^3 - 11x^2 + 36x + 36 = 0$ (B) $x^3 - 11x^2 + 36x - 36 = 0$
(C) $x^3 + 11x^2 - 36x + 36 = 0$ (D) $x^3 - 11x^2 - 36x + 36 = 0$

3. If α, β, γ are the roots of the equation $x^3 + ax - b = 0$, then $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 =$
(A) a^2 (B) $a^2 + b$ (C) $a^2 - 2b$ (D) $a^2 - b$

4. The numbers of positive and negative roots of the equation $x^3 + x^2 + x - 1 = 0$ are respectively :
(A) 1, 1 (B) 1, 2
(C) 2, 1 (D) 2, 2

5. How many roots does the equation $z^4 + z^2 + 1 = 0$ have in $|z| < 1$?
(A) 4 (B) 3 (C) 2 (D) 0

6. If 'a' and 'b' are the supremum and infimum of the set $S = \left\{\frac{1}{n}, n \in N\right\}$, then :
(A) $a \in S, b \in S$ (B) $a \in S, b \notin S$
(C) $a \notin S, b \in S$ (D) $a \notin S, b \notin S$

7. If Q, Q^c and R are the sets of rational, irrational and real numbers respectively, then the set Q together with the set of its limit points equals :
(A) Q (B) Q^c (C) R (D) ϕ

8. Which of the following is true for the sequences $\{a_n\}$ and $\{b_n\}$, where
$$a_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots\ldots + \frac{1}{n!} \text{ and } b_n = \frac{(-1)^n}{n}, n = 1, 2, 3 \ldots\ldots ?$$
(A) $\{a_n\}$ is convergent, $\{b_n\}$ is divergent
(B) $\{b_n\}$ is convergent, $\{a_n\}$ is divergent
(C) Both are convergent
(D) Both are divergent

9. At which point x in [–1, 1] is the tangent to the curve $y = \sqrt{1 - x^2}$ parallel to the x-axis ?

(A) $x = -1$ (B) $x = 1$

(C) $x = 0$ (D) None of the above

10. If f(x) is continuous on [a, b], then which of the following is not true in general ?

(A) $\int_a^b f(x)\,dx = \lambda(b - a)$ for some number λ lying between the bounds of f(x)

(B) $\int_a^b f(x)\,dx = (b - a)\,f(c)$ for some number c lying between a and b

(C) $\left| \int_a^b f(x)\,dx \right| \leq k\,|b - a|$ for some number k such that

$|f(x)| \leq k, \forall x \in [a, b]$

(D) $\int_a^b f(x)\,dx = 0$ if $f(x) \geq 0, \forall x \in [a, b]$

11. Which of the following is true for the function

$f(x) = 1$, x rational

$= 0$, x irrational

in [a, b] ?

(A) $\int_a^b f(x)\,dx = 0$

(B) $\int_a^b f(x)\,dx = 1$

(C) f(x) is not R-integrable over [a, b]

(D) None of the above

12. What is the value of $\int_{-1}^{1} |x|\,dx$?

(A) 1 (B) –1 (C) 2 (D) 0

13. If [x] denotes the greatest integer function not greater than x, then $\int_0^3 [x]\,dx =$

(A) 0 (B) 2 (C) 3 (D) 1

14. $\text{Lim}_{(x,y)\to(0,0)} \dfrac{xy(x^2-y^2)}{x^2+y^2} =$

(A) 0 (B) 1 (C) 2 (D) –1

15. If $f(x, y) = 2x^4 - 3x^2y + y^2$, then f(x, y) has :
(A) a maximum at (0, 0)
(B) a minimum at (0, 0)
(C) neither a maximum nor a minimum at (0, 0)
(D) none of the above properties

16. If $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\,dx$, then $\Gamma\left(\frac{3}{2}\right) =$

(A) $\sqrt{\pi}$ (B) $\dfrac{\sqrt{\pi}}{2}$ (C) $\sqrt{2\pi}$ (D) $\sqrt{\dfrac{\pi}{2}}$

17. What is the value of $\iint_E e^{\frac{y}{x}}\,dxdy$, where E is the triangle formed by the straight lines y = x, y = 0 and x = 1 ?

(A) e – 1 (B) $\dfrac{e}{2}$ (C) $\dfrac{1-e}{2}$ (D) $\dfrac{e-1}{2}$

18. If a function $f : [0, 1] \to R$ is defined by $f(x) = x^2$, then f is :
(A) one-one (B) onto
(C) one-one and onto (D) many-one

19. If $G = \{z \in C, z^n = 1\}$, then G is a multiplicative group which is :
(A) Abelian (B) non-abelian
(C) of prime order (D) without any subgroups

20. If the number of left cosets of a subgroup H of a group G with identity e, then :
(A) H = {e} (B) H = G
(C) H is normal in G (D) G is simple

21. If G is a group and $f : G \to G$ is defined by $f(x) = x^{-1} ax$, $\forall x \in G$ for some fixed element $a \in G$, then f is :

(A) a homomorphism on G
(B) an isomorphism on G
(C) an automorphism on G
(D) all the above

22. Which of the following is true for any two subrings S_1 and S_2 of a ring R ?

(A) $S_1 \cap S_2 = \Phi$
(B) $S_1 \cap S_2 \neq \Phi$
(C) $S_1 \cup S_2 = R$
(D) $S_1 \subset S_2$

23. Which of the following is both a prime and a maximal ideal in the ring of intergers ?

(A) (4)
(B) (6)
(C) (2)
(D) (15)

24. If X is a finite dimensional vector space and X′ its dual space, then :

(A) $\dim(X') < \dim(X)$
(B) $\dim(X') > \dim(X)$
(C) $\dim(X') = \dim(X)$
(D) $\dim(X) < \dim(X')$

25. If T is a linear transformation from the vector space X_F to the vector space Y_F, then T is one-one if and only if :

(A) $T(x) = 0 \Rightarrow x = 0$
(B) $x = 0 \Rightarrow T(x) = 0$
(C) Neither (A) nor (B) holds
(D) None of the above holds

26. Which of the following is a point of discontinuity of the function :

$$f(0) = x \sin\frac{1}{x}, x \neq 0$$
$$= 0, x = 0 ?$$

(A) $x = 0$
(B) $x = 1$
(C) $x = 2$
(D) None of the above

27. What is the valuc of thc 'c' of the Lagrange's Mean Value Theorem for the function $f(x) = 2x^2 - 7x + 10$ in [2, 5] ?

(A) $c = \frac{7}{4}$
(B) $c = \frac{7}{2}$
(C) $c = \frac{2}{7}$
(D) $c = \frac{10}{7}$

28. If f″(x) exists and is continuous, then $\frac{f(x+h) + f(x-h) - 2f(x)}{h} =$

(A) f′(x)
(B) f(x)
(C) f″(x)
(D) None of the above

29. If $u = \lim_{x \to 0} \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}} + 1}$, then :

(A) $u = 0$ (B) $u = 1$

(C) $u = e$ (D) u does not exist

30. What is the value of $\lim_{x \to 0} \frac{e^x \sin x - x - x^2}{x^3}$?

(A) 0 (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) 1

31. The angle of intersection of the curves $r = a\theta$ and $r = \frac{a}{\theta}$ is equal to :

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

32. If $u = \tan^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(A) $\sin 2u$ (B) $\sin u$ (C) $\frac{1}{2}\sin 2u$ (D) $\frac{1}{2}\sin u$

33. The points of numerically maximum curvature on the curve $y = \log \sin x$ are given by :

(A) $x = \pm n\pi$ (B) $x = \pm(2n+1)\frac{\pi}{2}$

(C) $x = \pm\frac{n\pi}{2}$ (D) $x = \pm(2n+1)\pi$,

where $n = 0, 1, 2, 3$ .....

34. Which of the following pairs gives the modulus and amplitude of the complex number :

$1 - \cos\phi + i\sin\phi$?

(A) $2\sin\frac{\phi}{2}, \frac{\pi}{2}$ (B) $2\sin\frac{\phi}{2}, \frac{\pi}{2} - \frac{\phi}{2}$

(C) $\sin\frac{\phi}{2}, \frac{\pi}{2}$ (D) $\sin\frac{\phi}{2}, \frac{\pi}{2} - \frac{\phi}{2}$

35. If $\cos 2\theta + i\sin 2\theta = p$, $\cos 2\phi + i\sin 2\phi = q$, then $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} =$

(A) $\cos(\theta - \phi)$ (B) $2\cos(\theta - \phi)$ (C) $2i\sin(\theta - \phi)$ (D) $i\sin(0 - \phi)$

36. The real and imaginary parts of Log(–i) are respectively :

(A) $1, -\frac{\pi}{2}$ (B) $0, (4n-1)\pi$

(C) $1, (4n-1)\frac{\pi}{2}$ (D) $0, (4n-1)\frac{\pi}{2}$

37. What is the sum to n terms of the series

$$\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \ldots\ldots\ldots\ldots ?$$

(A) $\tan^{-1}\frac{n}{n+1}$ (B) $\tan^{-1}\frac{n+1}{n}$

(C) $\tan^{-1}\frac{n}{n+2}$ (D) $\tan^{-1}\frac{n+2}{n}$

38. The coordinates of the foci and eccentricity of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ are respectively :

(A) $(0, \pm\sqrt{7}), \frac{\sqrt{7}}{4}$ (B) $(\pm\sqrt{7}, 0), \frac{\sqrt{7}}{4}$

(C) $(\pm\sqrt{7}, 0), \sqrt{7}$ (D) $(0, \pm\sqrt{7}), \sqrt{7}$

39. Which of the following is the equation of the normal to the rectangular hyperbola $x^2 - y^2 = a^2$ at the point (a sec θ, b tan θ) ?

(A) x sec 0 – y tan θ = a tan θ sec θ
(B) x tan 0 + y sec 0 = a tan θ sec θ
(C) x tan 0 + y scc 0 = 2a tan θ sec θ
(D) x sec 0 – y tan θ = 2a tan θ sec θ

40. What is the centre of the conic

$36x^2 + 24xy + 29y^2 - 72x + 126y + 81 = 0$?

(A) (2, 3) (B) (3, 2) (C) (3, –2) (D) (2, –3)

41. The image of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$ is :

(A) (2, –1, 1) (B) (1, 3, 4) (C) (–1, 4, 3) (D) (–3, 5, 2)

42. The distance of the centre of the sphere

$x^2 + y^2 + z^2 - 2y - 4z = 11$

from the yz-plane is :

(A) 0 (B) 1 (C) 2 (D) 11

43. The semi-vertical angle of a right circular cone having 3 mutually perpendicular generators is :

(A) $\tan^{-1}\frac{1}{\sqrt{2}}$ (B) $\tan^{-1}\sqrt{2}$ (C) $\tan^{-1}\sqrt{3}$ (D) $\tan^{-1}\frac{1}{\sqrt{3}}$

44. The locus of the tangents to a sphere which are parallel to a given line is :

(A) a sphere (B) a cone (C) a cylinder (D) a circle

45. Which of the following is the condition for the plane $lx + my + nz = p$ to touch the conicoid $ax^2 + by^2 + cz^2 = 1$?

(A) $al^2 + bm^2 + cn^2 = p^2$ (B) $a^2l^2 + b^2m^2 + c^2n^2 = p^2$

(C) $a^2l + b^2m + c^2n = p^2$ (D) $\frac{\ell^2}{a} + \frac{m^2}{b} + \frac{n^2}{c} = p^2$

46. What is the value of $\int \frac{xe^x\,dx}{(x+1)^2}$?

(A) $\frac{e^x}{(x+1)^2} + c$ (B) $\frac{e^x}{x^2+1} + c$

(C) $\frac{e^x}{x+1} + c$ (D) None of the above

47. Which of the following is the value of $\int_0^{\frac{\pi}{2}} \sin^5 x\,dx$?

(A) $\frac{4}{5}$ (B) $\frac{8}{15}$

(C) $\frac{3}{4}$ (D) $\frac{15}{8}$

48. Find the value of $\lim_{x\to 0} \frac{\int_0^x \sin t\,dt}{x^2}$.

(A) $\sin x$ (B) $\cos x$

(C) $\frac{1}{3}$ (D) None of the above

49. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-\hat{k}$, then $\vec{a}.\vec{b}\times\vec{c}$ and $\vec{a}\times(\vec{b}\times\vec{c})$ are respectively equal to :

(A) $4, -2\hat{j}+2\hat{k}$ (B) $4, 2\hat{i}-2\hat{k}$

(C) $4, 2\hat{i}-2\hat{j}$ (D) $4, 2\hat{j}-2\hat{k}$

50. The pair giving the degree and order of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=\left(\frac{d^2y}{dx^2}\right)^2 \text{ is :}$$

(A) (3, 2) (B) (2, 2) (C) (2, 3) (D) (3, 3)

51. Which of the following is the general solution of the differential equation $(1+x^2)\frac{dy}{dx}+y=e^{\tan^{-1}x}$?

(A) $y=\frac{1}{2}e^{-\tan^{-1}x}+ce^{\tan^{-1}x}$ (B) $y=\frac{1}{2}e^{\tan^{-1}x}+c^{-\tan^{-1}x}$

(C) $y=\frac{1}{2}e^{2\tan^{-1}x}+ce^{-2\tan^{-1}x}$ (D) $y=\frac{1}{2}e^{-2\tan^{-1}x}+ce^{2\tan^{-1}x}$

52. The general solution of the differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2e^{3x}$ is :

(A) $y=(c_1+c_2x)e^x+\frac{e^{3x}}{8}(2x^2-4x+3)$

(B) $y=(c_1+c_2x)e^x$

(C) $y=(c_1+c_2x+c_3x^2)e^x$

(D) $y=(c_1+c_2x)e^x+\frac{e^{3x}}{8}(x^2-x+2)$

53. The general solution of the differential equation $x\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^2-y=0$ is :

(A) $y=cx+c^2$ (B) $y=cx-\frac{1}{c^2}$

(C) $xy=c$ (D) None of the above

54. If A is a skew-symmetric matrix of order n, then :

(A) $|A|=n$ (B) $|A|=1$

(C) $|A|=0$ (D) None of the above

55. If A′ is the transpose of a square matrix A and $P=\frac{A+A'}{2}, Q=\frac{A-A'}{2}$, then :

(A) P is symmetric, Q is skew-symmetric
(B) P and Q are both symmetric
(C) Q is symmetric, P is skew-symmetric
(D) Neither P nor Q is symmetric

56. If $A=\begin{bmatrix}5 & 6 & 7\\ 4 & 3 & 2\\ 1 & 8 & -9\end{bmatrix}$, $B=\begin{bmatrix}\lambda & 6 & 7\\ 4 & \mu & 2\\ 1 & 8 & \nu\end{bmatrix}$ and $A+2B=\begin{bmatrix}1 & 18 & 21\\ 12 & 5 & 6\\ 3 & 24 & 5\end{bmatrix}$, then $(\lambda, \mu, \nu) =$

(A) (–2, 1, 7)
(B) (–1, –2, 7)
(C) (2, –1, 7)
(D) (7, 1, 2)

57. If A is a square matrix of order 3 such that $|A| = k$, then $|2A| =$

(A) 2k
(B) 4k
(C) 6k
(D) 8k

58. If $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$, for what value of m and n is the characteristic

$-x^3 + mx^2 + nx + 4 = 0$

the characteristic equation of A ?

(A) m = –6, n = 9
(B) m = 6, n = –9
(C) m = 9, n = 6
(D) m = –9, n = 6

59. What is the rank of the matrix $A=\begin{bmatrix}2 & 3 & -1 & -1\\ 1 & -1 & -2 & -4\\ 3 & 1 & 3 & -2\\ 6 & 3 & 0 & -7\end{bmatrix}$?

(A) 4 (B) 3 (C) 2 (D) 1

60. The vectors [2 3 –1 –1], [1 –1 –2 –4], [3 1 3 –2] and [6 3 0 –7] :

(A) are linearly independent
(B) are linearly dependent
(C) form a basis for the vector space of all row matrices of order 4
(D) satisfy none of the above properties

1. Which of the following are respectively the infimum and supremum of the set $\left\{\frac{(-1)^n}{n}, n \in N\right\}$?

   (A) 0, 1 (B) $-1, 1$

   (C) $-1, \frac{1}{2}$ (D) $\frac{1}{2}, 1$

2. Which of the following is the value of $\lim a_n$ where $a_n = 1 + (-1)^n$, $n \in N$?

   (A) 0 (B) 1

   (C) -1 (D) 2

3. For what value of S_n is the sequence $\{S_n\}$ convergent ?

   (A) $S_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$

   (B) $S_n = 1 + \frac{1}{3} + \frac{1}{5} + ... + \frac{1}{2n-1}$

   (C) $S_n = 1 + \frac{1}{4} + \frac{1}{7} + ... + \frac{1}{3n-2}$

   (D) $S_n = 1 + \frac{1}{4} + \frac{1}{9} + ... + \frac{1}{n^2}$

4. The image of a closed interval under a continuous function is :

   (A) a closed interval (B) an open interval

   (C) a semi-open interval (D) a semi-closed interval

5. The upper and lower Riemann integrals of the functic $f(x) = 0$, x rational
   $= 1$, x irrational

   on any interval [a, b] are respectively equal to :

   (A) $0, b - a$ (B) a, b

   (C) $b - a, b + a$ (D) $b - a, 0$

6. What is the value of the R-intergral $\int_{-1}^{1} |x|\, dx$?

(A) 0 (B) -1

(C) 1 (D) 2

7. Which of the following is not true for the function

$f(x) = 1$, x rational

$= -1$, x irrational

defined on [a, b] ?

(A) $\int_a^b f(x)\, dx = a - b$

(B) $\int_a^b f(x)\, dx = b - a$

(C) f(x) is R-integrable over [a, b]

(D) f(x) is not R-integrable over [a, b]

8. If a function f(x) has an infinite number of discontinuities in [a, b] then :

(A) f(x) is not R-integrable over [a, b]

(B) f(x) is R-integrable over [a, b]

(C) f(x) may or may not be R-integrable over [a, b]

(D) None of the above holds

9. If $f(x,y) = \left(\frac{y-x}{y+x}\right)\left(\frac{1+x}{1+y}\right)$, then $\lim_{(x,y)\to(0,0)} f(x,y)$:

(A) $= 0$ (B) $= 1$

(C) $= -1$ (D) Does not exist

10. For the function :

$$f(x,y) = \frac{xy(x^2 - y^2)}{x^2 + y^2}, (x, y) \neq (0, 0)$$

$= 0, (x, y) = (0, 0)$

(A) $f_{xy} = f_{yx}$ (B) $f_{xy} \neq f_{yx}$

(C) f_{xy} and f_{yx} do not exist (D) None of the above holds

11. The value of $\iint_A (x + y)\,dx\,dy$ where $A = \{(x, y), 0 < x < 1, 0 < y < 1\}$ is :

(A) 0 (B) 1

(C) $\frac{1}{2}$ (D) 2

12. If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r \cos\theta$, then

$$\frac{\partial (x,y,z)}{\partial (r,\theta,\varphi)} =$$

(A) $r \sin\theta$ (B) $r \cos\theta$

(C) $r^2 \sin\theta$ (D) $r^2 \cos\theta$

13. If in a group G, $a^{-1} = a$, $\forall\ a \in G$, then G is :

(A) finite (B) infinite

(C) abelian (D) non-abelian

14. The intersection of two subgroups of finite index is :

(A) empty (B) of finite index

(C) normal (D) none of the above

15. If G is a group with center Z(G) and if $\frac{G}{Z(G)}$ is cyclic, then G is :

(A) finite (B) infinite

(C) abelian (D) non-abelian

16. The converse of Lagrange's theorem is true for :

(A) finite groups (B) abelian groups

(C) non-abelian groups (D) cyclic groups

17. In the ring {a + bi; a, b integers} with respect to usual addition and multiplication, the number of units is :

(A) 1 (B) 2

(C) 3 (D) 4

18. If R is a commutative ring with unity having no nontrivial ideals, then R is :

(A) a division ring
(B) an integral domain
(C) a field
(D) none of the above

19. Which of the following is a subspace of the vector space of complex numbers over the field of real numbers ?

(A) the set of integers
(B) the set of natural numbers
(C) the set of real numbers
(D) the set of irrational numbers

20. The dimension of the vector space of complex numbers over the real field is :

(A) 1
(B) 2
(C) 3
(D) 4

21. The function f(x) is defined by

$$f(x) = \frac{|x-4|}{x-4},\ x \neq 4$$
$$= 0,\ x = 4$$

what is the value of $\lim_{x \to 4} f(x)$?

(A) 0
(B) 1
(C) -1
(D) It does not exist

22. If $f(x) = \frac{\sin 2x}{x},\ x \neq 0$
$= 1,\ x = 0,$

then f(x) has :

(A) a removable discontinuity at $x = 0$
(B) a discontinuity of the first kind $x = 0$
(C) a discontinuity of the second kind at $x = 0$
(D) none of the above properties

23. Which of the following is the value of the 'C' of the Lagrange's Mean Value Theorem for the function $f(x) = \log x$ on $\left[\frac{1}{2}, 2\right]$?

(A) $3 \log 2$ (B) $\frac{2}{3 \log 2}$

(C) $\frac{3}{\log 2}$ (D) $\frac{3}{4 \log 2}$

24. If $f(x) = 0, x \leq 0$

$= x, x > 0,$

Then f(x) is :

(A) continuous and differentiable at $x = 0$

(B) differentiable but not continuous at $x = 0$

(C) continuous but not differentiable at $x = 0$

(D) neither continuous nor differentiable at $x = 0$

25. Which of the following is the value of $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$?

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$

(C) $-\frac{1}{3}$ (D) $-\frac{1}{2}$

26. The polar subtangent for the parabola $\frac{2a}{\gamma} = 1 - \cos\theta$ is equal to :

(A) $a \operatorname{cosec} \frac{\theta}{2}$ (B) $2a \operatorname{cosec} \frac{\theta}{2}$

(C) $a \operatorname{cosec} \theta$ (D) $2a \operatorname{cosec} \theta$

27. The radius of curvature at any point of the curve

$x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$

is equal to :

(A) $a \sin \frac{\theta}{2}$ (B) $4a \sin\theta$

(C) $4a \sin \frac{\theta}{2}$ (D) $4a \cos\theta$

28. What type of a double point is the origin on the curve $a^2y^2 = a^2x^2 - 4x^3$?

(A) a node
(B) a cusp
(C) a conjugate point
(D) none of the above

29. For any two complex numbers z_1 and z_2, $|z_1 + z_2|^2 + |z_1 - z_2|^2 - 2(|z_1|^2 + |z_2|^2) =$

(A) 0
(B) 1
(C) $|z_1|^2 + |z_2|^2$
(D) $|z_1|^2 - |z_2|^2$

30. What is the value of $\frac{\left(\sqrt{3}+i\right)^4}{\left(-1+i\sqrt{3}\right)^6}$?

(A) $\frac{1}{8} - i\frac{\sqrt{3}}{8}$
(B) $-\frac{1}{8} + i\frac{\sqrt{3}}{8}$
(C) $\frac{1}{8} + i\frac{\sqrt{3}}{8}$
(D) $-\frac{1}{8} - i\frac{\sqrt{3}}{8}$

31. What is the real part of $\text{Sin}^{-1}(\cos\theta + i\sin\theta)$?

(A) $\sin^{-1}\left(\sqrt{\sin\theta}\right)$
(B) $\cos^{-1}\left(\sqrt{\cos\theta}\right)$
(C) $\sin^{-1}\left(\sqrt{\cos\theta}\right)$
(D) $\cos^{-1}\left(\sqrt{\sin\theta}\right)$

32. $\log\left(\frac{a+bi}{a-bi}\right) =$:

(A) $\frac{\pi}{2}i$
(B) $i\tan^{-1}\left(\frac{b}{a}\right)$
(C) $2i\tan^{-1}\left(\frac{b}{a}\right)$
(D) $i\tan h^{-1}\left(\frac{b}{a}\right)$

33. The line $y = mx + c$ is a normal to the parabola $y^2 = 4ax$ for all values of m if :

(A) $c = am + \frac{a}{m}$
(B) $c^2 = a^2 + m^2$
(C) $c = \frac{a}{m}$
(D) $c = -2am - am^3$

34. The eccentricity and the distance between the foci of the ellipse $3x^2 + 4y^2 = 12$ are respectively equal to :

(A) 2 and $\frac{1}{2}$

(B) $\frac{1}{2}$ and 2

(C) $\frac{1}{3}$ and 3

(D) $\frac{1}{4}$ and 4

35. The equation of the diameter which is conjugate to the diameter $x = 2y$ of the hyperbola $16x^2 - 9y^2 = 144$ is :

(A) $y = 2x$

(B) $16x = 9y$

(C) $32x = 9y$

(D) $9x = 16y$

36. The equation of a straight line through the point (3, 1, –6) and parallel to each of the planes $x + y + 2z - 4 = 0$ and $2x - 3y + z + 5 = 0$ is :

(A) $\frac{x-3}{1} = \frac{y-1}{1} = \frac{z+6}{2}$

(B) $\frac{x-3}{2} = \frac{y-1}{-3} = \frac{z+6}{1}$

(C) $\frac{x-3}{3} = \frac{y-1}{7} = \frac{z+6}{-5}$

(D) $\frac{x-3}{7} = \frac{y-1}{3} = \frac{z+6}{-5}$

37. The spheres $a(x^2 + y^2 + z^2) + 2lx + 2my + 2nz + p = 0$ and $b(x^2 + y^2 + z^2) = k^2$ will cut orthogonally if :

(A) $l^2 + m^2 + n^2 = p^2$

(B) $al^2 + bm^2 + pn^2 = 0$

(C) $ak^2 = bp$

(D) $a^2 + b^2 = p^2$

38. The general equation of a cone which passes through the axes is :

(A) $ax^2 + by^2 + cz^2 = 0$

(B) $ax^2 + by^2 + cz^2 = 1$

(C) $fyz + gzx + hxy = 0$

(D) $fyz + gzx + hxy = 1$

39. The locus of the points of intersection of three mutually perpendicular tangent planes to the central conicoid $ax^2 + by^2 + cz^2 = 1$ is a :

(A) circle
(B) sphere
(C) cone
(D) cylinder

40. The number of normals that can be drawn to the conicoid $ax^2 + by^2 + cz^2 = 1$ from a given point in general is :

(A) six
(B) five
(C) three
(D) two

41. What is the value of $\int_a^b \frac{\log x}{x}\,dx$?

(A) $2 \log\left(\frac{b}{a}\right) \log(ab)$
(B) $2 \log\left(\frac{a}{b}\right) \log(a\,b)$
(C) $\frac{1}{2} \log\left(\frac{b}{a}\right) \log(a\,b)$
(D) $\frac{1}{2} \log\left(\frac{a}{b}\right) \log(a\,b)$

42. Which of the following is the value of the integral $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) dx$?

(A) $2x \tan^{-1} x - \log(1 + x^2) + c$
(B) $2x \tan^{-1} x + \log(1 + x^2) + c$
(C) $x \tan^{-1} x - \frac{1}{2} \log(1 + x^2) + c$
(D) $x \tan^{-1} x + \frac{1}{2} \log(1 + x^2) + c$

43. For any three vectors $\vec{a}, \vec{b}, \vec{c}$, what is the value of

$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b})$?

(A) 3
(B) $\vec{a} \cdot \vec{b} \times \vec{c}$
(C) $\vec{a} + \vec{b} + \vec{c}$
(D) 0

44. $\nabla^2\gamma$ =

(A) $\frac{1}{\gamma}$ (B) $\frac{2}{\gamma}$

(C) 2γ (D) γ^2

45. Which of the following is the solution of the differential equation

$\frac{dy}{dx} + 3y = e^{2x}$?

(A) $y = (2x + c)\, e^{-x^2}$ (B) $y = \frac{1}{5}e^{2x} + ce^{-3x}$

(C) $y = cx + 2e^{-x^2}$ (D) $y = \frac{1}{5}e^{-3x} + ce^{2x}$

46. For what value of 'a' and 'b' is the function $a\,e^{bx}$ particular integral of the differential equation

$\frac{d^2y}{dx^2} + 13\frac{dy}{dx} + 42y = 112e^x$?

(A) $a = 2, b = 1$ (B) $a = 6, b = -1$

(C) $a = -4, b = 2$ (D) $a = 1, b = 2$

47. Which of the following is the value of $\frac{1}{D^2+4}(\sin 2x)$?

(A) $\frac{1}{4}x \sin 2x$ (B) $-\frac{1}{4}x \sin 2x$

(C) $-\frac{1}{4}x \cos 2x$ (D) $\frac{1}{4}x \cos 2x$

48. Which of the following is the complete primitive of the differential equation

$p^2 + p - 6 = 0$, where $p = \frac{dy}{dx}$?

(A) $(y - 2x - c)(y + 3x - c) = 0$

(B) $(y - 2x - c_1)(y + 3x - c_2)$

(C) $(y - 3x - c_1)(y + 2x - c_2) = 0$

(D) $(y - 3x - c)(y + 2x - c) = 0$

49. If $D = \text{diag}(\alpha_1, \alpha_2, \ldots\ldots, \alpha_n)$ is a diagonal matrix of order n, then $D^p =$

(A) $\text{diag}(\alpha_1, \alpha_2, \ldots\ldots, \alpha_n)$

(B) $\text{diag}(p\alpha_1, p\alpha_2, \ldots\ldots, p\alpha_n)$

(C) $\text{diag}(\alpha_1 + p, \alpha_2 + p, \ldots\ldots, \alpha_n + p)$

(D) $\text{diag}(\alpha_1^p, \alpha_2^p, \ldots\ldots, \alpha_n^p)$

50. If A is a Skew-Hermitian matrix then the diagonal elements of A are all :

(A) zeros

(B) purely imaginary

(C) zeros or real

(D) zeros or purely imaginary

51. If X and Y are inversible matrices of the same order, then X·Y is inversible if and only if:

(A) $X = Y$

(B) $X^2 = Y^2$

(C) $XY = YX$

(D) $X^2 + Y^2 = 0$

52. How many square matrices A and B are there such that $AB - BA = I$?

(A) finite

(B) infinite

(C) finite or infinite

(D) none

53. For what values of α nd β is the equation $\lambda^3 - 6\lambda^2 + \alpha\lambda - \beta = 0$, the characteristics equation of the matrix $\begin{bmatrix} 1 & 2 & 5 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix}$?

(A) $\alpha = 6, \beta = 11$

(B) $\alpha = 11, \beta = 6$

(C) $\alpha = 3, \beta = 5$

(D) $\alpha = 2, \beta = 3$

54. What is the rank of the matrix $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 5 & 6 \\ 1 & 3 & 2 & 1 \end{bmatrix}$?

(A) 4

(B) 3

(C) 2

(D) 1

55. If A is a 4×5 matrix with rank 3, then the number of linearly independent solutions of the equation $AX = 0$ is :

(A) 4 (B) 3

(C) 2 (D) 1

56. If $C_1, C_2, \ldots\ldots C_n$ are the linearly dependent columns of a $m \times n$ matrix A of rank r, then :

(A) $r < n$ (B) $r > n$

(C) $r = n$ (D) None of the above holds

57. If $a_1, a_2, \ldots\ldots, a_n$ are the roots of the equation $x^n + nax - b = 0$, then $(a_1 - a_2)(a_1 - a_3) \ldots\ldots (a_1 - a_n) =$

(A) na_1 (B) na_1^{n-1}

(C) $(n-1)a$ (D) $n(a_1^{n-1} + a)$

58. If the roots of the equation $x^3 - px^2 + qx - r = 0$ are in H.P., then the mean root is equal to :

(A) $\frac{3p}{q}$ (B) $\frac{3q}{p}$

(C) $\frac{3r}{q}$ (D) $\frac{3q}{r}$

59. Which of the following is the equation whose roots are the cubes of the roots of the equation $x^3 + 3x^2 + 2 = 0$?

(A) $y^3 + 33y^2 + 12y + 8 = 0$ (B) $y^3 + 12y^2 + 33y + 8 = 0$

(C) $y^3 + 8y^2 + 12y + 33 = 0$ (D) $y^3 + 33y^2 + 8y + 12 = 0$

60. What is the minimum number of imaginary roots which the equation $2x^7 - x^4 + 4x^3 - 5 = 0$ can possess ?

(A) 2 (B) 3

(C) 4 (D) 5

Mathematics - 2010

M. Sc. Mathematics

1. Which of the following is true ?

   I. As $x \to 0$, the infinitesimal $\log(1+x)$ is of higher order than the infinitesimal x

   II. As $x \to 0$, the infinitesimal $\sqrt{x(1-x)}$ is of lower order than the infinitesimal x

   (a) I only (b) II only

   (c) Both I and II (d) None of the above

2. In the Lagrange's Mean Value theorem, what is the value of C if $f(x) = x^3 - 3x^2 + 2x$, $a = 0$ and $b = ½$?

   (a) $1 - \frac{\sqrt{3}}{3}$ (b) $1 + \frac{\sqrt{21}}{6}$

   (c) $1 - \frac{\sqrt{21}}{6}$ (d) None of the above

3. What is the coefficient of x^3 in the power series expansion of the function $\sin^{-1} x$:

   (a) $\frac{1}{6}$ (b) $\frac{1}{3}$

   (c) $\frac{1}{2}$ (d) None of the above

4. Which of the following is the value of the limit :

   $\lim_{x \to 0} \frac{(3\tan x - 3x - x^3 + x^4 \sin x)}{x^5}$?

   (a) Non - existent (b) $\frac{7}{5}$

   (c) $\frac{2}{5}$ (d) None of the above

5. If P is the radius of curvature at any point P on the parabola $y^2 = 8x$ and S is its focus, then what is the value of $\frac{P^2}{(SP)^3}$?

   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$

   (c) 4 (d) 2

6. Consider the curve $y^2 = (x - a)(x - b)(x - c)$, $0 < a < b < c$. Then which of the following is true ?

I. The curve has no double points

II. A part of the curve lies to the left of the line $x = a$

(a) None (b) Both I and II

(c) I only (d) II only

7. If Z_1 and Z_2 are two complex numbers such that $|Z_1|^2 + |Z_2|^2 = 5$, then what is the value of $\sqrt{\{|Z_1 + 2Z_2|^2 + |2Z_1 - Z_2|^2\}}$?

(a) It cannot be determined by the given information

(b) $\sqrt{10}$

(c) 4

(d) 5

8. If $\sin\alpha + \sin\beta + \sin\gamma = 0 = \cos\alpha + \cos\beta + \cos\gamma$, then which of the following is the value of M + N where $M = \sin(\alpha + \beta) + \sin(\beta + \gamma) + \sin(\gamma + \alpha)$ and $N = \frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos(\alpha + \beta + \gamma)}$?

(a) 3 (b) 1

(c) 0 (d) None of the above

9. Which of the following statements is true :

I. The sum of the r^{th} powers of the n^{th} roots of unity vanishes if r is a multiple of n

II. The product of the n^{th} roots of unity is $(-1)n^{-1}$

(a) I only (b) II only

(c) Both I and II (d) None of the above

10. For what value of $\psi(\theta)$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, $\log(1 + e^{i2\theta}) - i\theta = \psi(\theta)$?

(a) $\log(2\cos\theta)$ (b) $i\theta$

(c) $\log(2\sin\theta) + i\theta$ (d) None of the above

11. What is the value of $\tan^{-1}1 + 2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$?

(a) π (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{2}$ (d) None of the above

12. An equilateral triangle is inscribed in the parabola $y^2 = 2x$, one of whose vertex is the vertex of the parabola. If a is the length of its sides, what is the value of a ?

(a) $4\sqrt{3}$ (b) $2\sqrt{3}$

(c) $\sqrt{3}$ (d) None of the above

13. If e is the eccentricity and l is the length of the latus rectum of the ellipse $9x^2 + 25y^2 = 225$, what is the value of the pair (e, l) ?

(a) $\left(\frac{4}{3}, \frac{50}{9}\right)$ (b) $\left(\frac{4}{3}, \frac{18}{5}\right)$

(c) $\left(\frac{4}{5}, \frac{50}{3}\right)$ (d) $\left(\frac{4}{5}, \frac{18}{5}\right)$

14. Which of the following is the equation of the hyperbola whose directrix is $2x + y = 1$, focus $(1, 1)$ and eccentricity $\sqrt{3}$?

(a) $x^2 + y^2 - 2x - 2y - 4 = 0$

(b) $5x^2 + 5y^2 + 4x - 2y - 1 = 0$

(c) $7x^2 + 2y^2 - 12xy - 10x + 8y + 5 = 0$

(d) $7x^2 - 2y^2 + 12xy - 2x + 4y - 7 = 0$

15. For which of the following values of the pair (α, β), $\alpha x + \beta y - 3z = 5$ represents the equation of the plane passing through the points $(1, 2, 1)$ and $(-2, 2, -1)$?

(a) $(3, 2)$ (b) $(2, 3)$

(c) $(-3, 1)$ (d) None of the above

16. If the axes are rectangular and O is the origin, what is the equation of the plane through the point P $(2, 3, -1)$ at right angles to OP ?

(a) $3x + 2y - z = 11$ (b) $3x - 2y + 4z + 4 = 0$

(c) $2x + 3y - z = 14$ (d) None of the above

17. For which of the following values of the pair (α, β), $\frac{x-3}{\alpha} = \frac{y+1}{\beta} = \frac{z-11}{4}$ is the equation of the perpendicular from $(3, -1, 11)$ to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$?

(a) $(1, -6)$ (b) $(6, -1)$

(c) $(4, 5)$ (d) None of the above

18. What is the radius of the circular section of the sphere $x^2 + y^2 + z^2 = 41$ by the plane $2x + 3y + z = 5\sqrt{14}$?

(a) $\sqrt{41}$ (b) 5

(c) $\sqrt{14}$ (d) 4

19. If $a^{-1} + b^{-1} + c^{-1} = 0$, what is the angle between the lines of intersection given by $x + y + z = 0$ and the cone $ayz + bzx + cxy = 0$?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) None of the above

20. What are the equations of the tangent planes of the conicoid $2x^2 - 6y^2 + 3z^2 = 5$, which pass through the line $x + 9y - 3z = 0, 3x - 3y + 6z - 5 = 0$?

(a) $4x - 6y + z = 5$ and $4x - 9y + 2z = 5$

(b) $4x + 6y - 3z = 5$ and $5x + 15y - z = 5$

(c) $4x + 6y + 3z = 5$ and $2x - 12y + 9z = 5$

(d) None of the above

21. What is the value of the integral $\int_0^1 \left(\frac{1-x^2}{1+x^2}\right) dx$?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2} + 1$

(c) $\frac{\pi}{2} - 1$ (d) None of the above

22. Which of the following is the value of the integral $\int_0^{\frac{\pi}{2}} \sin^3 x \, dx$?

(a) $\frac{2}{3}$ (b) $\frac{8}{3}$

(c) $\frac{10}{3}$ (d) None of the above

23. What is the value of the integral $\int_0^1 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{1-x}}dx$?

(a) 0 (b) $\frac{1}{2}$

(c) 2 (d) None of the above

24. What is the entire length of the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4b^{\frac{2}{3}}$?

(a) 6b (b) 12b

(c) 24b (d) 48b

25. Three vectors $\vec{a} = 12\vec{i} + 4\vec{j} + 3\vec{k}, \vec{b} = 8\vec{i} - 12\vec{j} - 9\vec{k}$ and $\vec{c} = 33\vec{i} - 4\vec{j} - 24\vec{k}$ define a parallelopiped. What is its volume ?

(a) 3696 (b) 5915

(c) 9061 (d) None of the above

26. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a'}, \vec{b'}, \vec{c'}$ form a reciprocal system of vectors, then which of the following is true ?

I. $\vec{a}.\vec{a'} + \vec{b}.\vec{b'} + \vec{c}.\vec{c'} = 1$ II. $\vec{a} \times \vec{a'} + \vec{b} \times \vec{b'} + \vec{c} \times \vec{c'} = 0$

(a) Both I and II (b) I only

(c) II only (d) None of the above

27. The differential equation $(1+x^2)\left(\frac{dy}{dx}\right) - 4x^2 \cos^2 y + x \sin 2y = 0$ can be reduced to the linear form by the transformation z =

(a) $1 + x^2$ (b) $\cos^2 y$

(c) $\sin^2 y$ (d) $\tan y$

28. What is the value of $\left[\frac{1}{(D+3)^2}\right](x^2+1)$ where $D = \frac{d}{dx}$?

(a) $4(x^2+3)^2$ (b) $\frac{1}{27}(3x^2 - 4x + 5)$

(c) $\frac{1}{27}(3x^2 - x + 2)$ (d) None of the above

29. Which of the following is the solution of the differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^3=0$?

(a) $y = x^2 + C_1x + C_2$ (b) $\text{Sin}(C_1 - y) = C_2e^{-x}$

(c) $\sin(y + C_1) = C_2x$ (d) None of the above

30. For what value of the pair (α, β) Lagendre polynomial $P_4(x)=\frac{1}{8}(\alpha x^4-30x^2+\beta)$?

(a) (5, 1) (b) (25, 3)

(c) (35, 3) (d) None of the above

31. Let Q be a skew - symmetric matrix of order 3. For which of the following values of n, the matrix Q^n is a symmetric matrix of order 3 :

(a) 6 (b) 7

(c) 9 (d) None of the above

32. Let $P=\begin{bmatrix}2 & -1 & 3\\ 0 & 2 & 0\\ 2 & 1 & 1\end{bmatrix}$ and $Q=\begin{bmatrix}2 & 4 & -6\\ 0 & \alpha & \beta\\ -4 & -4 & 4\end{bmatrix}$. For which of the following values of the pair (α, β), Q = (adj P) ?

(a) (4, 4) (b) (–4, 4)

(c) (0,– 4) (d) (–4, 0)

33. What is the rank of the matrix P^3, where $P=\begin{bmatrix}1 & 1 & 3\\ 5 & 2 & 6\\ -2 & -1 & -3\end{bmatrix}$?

(a) 3 (b) 2

(c) 1 (d) 0

34. Let $P = [p_{ij}]$ be a square matrix of order $n \geq 2$. Then which of the following is true ?

I. adj (kP) = k (adj P), where k is a scalar

II. $|\text{adj } P| = |P|^{n-1}$, if $|P| \neq 0$

(a) I only (b) II only

(c) Both I and II (d) None of the above

35. For what value of λ, the column vectors $X_1 = \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}, X_2 = \begin{bmatrix} 2 \\ \lambda \\ -1 \end{bmatrix}, X_3 = \begin{bmatrix} 10 \\ 11 \\ 2 \end{bmatrix}$ are linearly dependent?

(a) -1 (b) 1

(c) 3 (d) None of the above

36. How many linearly independent solutions does the equation $6x - 5y + 4z - 3t = 21$ have?

(a) Three (b) Two

(c) One (d) None of the above

37. For what value of p, the polynomial $2x^4 - 7x^3 + p^2x + 15$ is divisible by $x - 3$?

(a) ± 2 (b) $\pm\sqrt{14}$

(c) 3 (d) None of the above

38. For which of the following values of k, the equation $x^3 - 9x^2 + 24x + k = 0$, has a multiple root?

(a) -8 (b) -16

(c) 32 (d) None of the above

39. The equation $x^n - nx + n - 1 = 0, n \geq 2$, is satisfied by $x = 1$. What is the multiplicity of this root?

(a) more than 3 (b) 3

(c) 2 (d) 1

40. How many roots does the equation $Z^4 - 2Z^3 + 3Z^2 - 4Z + 5 = 0$ have in the circle $|Z| \leq 1$?

(a) None (b) One

(c) Two (d) Four

41. Which of the following are true?

I. Every finite set of numbers is bounded

II. The set N of natural numbers is bounded above but not bounded below

III. The set Q of rational numbers is not bounded

(a) All I, II and III (b) I and II only

(c) II and III only (d) III and I only

42. Which of the following statements is true ?

I. The set $\left\{\frac{1}{n}; n = 1, 2, 3, ...\right\}$ has no limit point.

II. Every point of the set Z of all integers is a limit point of Z.

(a) Both I and II
(b) I only
(c) II only
(d) None of the above

43. The sequence $\{b_n\}$ where $b_n = \frac{1}{n}\left\{1 + 2^{\frac{1}{2}} + 3^{\frac{1}{3}} + ... + n^{\frac{1}{n}}\right\}$ converges to :

(a) 0
(b) 1
(c) e
(d) None of the above

44. Which of the following is true ?

I. The series $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$ is convergent.

II. The series $\sum_{n=3}^{\infty} \frac{1}{n \log n (\log \log n)^2}$ is divergent.

(a) None
(b) Both I and II
(c) I only
(d) II only

45. If a function f is derivable on a closed interval [a, b] with $f'(a) \neq f'(b)$ and if k is a number lying between $f'(a)$ and $f'(b)$ then there exist at least on point C between a and b such that :

(a) $f(C) = k$
(b) $f'(k) = C$
(c) $f'(C) = k$
(d) None of the above

46. What is the value of the integral $\int_0^2 x[x]\,dx$, where [x] is the largest integer less or equal to x ?

(a) $\frac{3}{2}$
(b) $\frac{7}{3}$
(c) $\frac{8}{3}$
(d) None of the above

47. What is the value of the simultaneous limit $\lim_{(x,y)\to(2,1)} \frac{\sin^{-1}(xy - 2)}{\tan^{-1}(3xy - 6)}$?

(a) 0
(b) 1
(c) $\frac{1}{2}$
(d) $\frac{1}{3}$

48. If V is a function of two variables x and y then for what value of the pair (α, β), $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}-\frac{\partial^2 V}{\partial r^2}=\alpha\frac{\partial^2 V}{\partial \theta^2}+\beta\frac{\partial V}{\partial r}$ where $x = r\cos\theta$ and $y = r\sin\theta$:

(a) $\left(\frac{1}{r}, \frac{1}{r^2}\right)$ (b) $\left(\frac{1}{r^2}, \frac{1}{r}\right)$

(c) $\left(1, \frac{1}{r}\right)$ (d) None of the above

49. If $xyz = 4(x + y + z)$, what is the minimum value of $yz + zx + xy$?

(a) 0 (b) 18

(c) 36 (d) None of the above

50. Which of the following is the value of the double integral $\iint_E \sqrt{x^2 + y^2}\,dx\,dy$ where E is the region in the xy - plane bounded by $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$?

(a) $\frac{38\pi}{3}$ (b) 5π

(c) $\frac{14\pi}{3}$ (d) None of the above

51. What is the value of the triple integral $\iiint_E xyz\,dx\,dy\,dz$ where E is a domain bounded by $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$?

(a) $\frac{1}{6}$ (b) $\frac{1}{24}$

(c) $\frac{1}{120}$ (d) $\frac{1}{720}$

52. Which of the following is true on Z, the set of all integers ?

I. The binary operation * on Z defined by $a * b = a^2 + b^2$ is commutative as well associative.

II. The binary operation * on Z given by $a * b = 2a + b$ is neither commutative nor associative.

(a) Both I and II (b) I only

(c) II only (d) None of the above

53. Which of the following sets together with the respective binary operations defined on them are groups ?

I. The set $G = \{2^n; n \in Z\}$ with the operation of ordinary multiplication.

II. The set Q of all rationals with the operation * defined by $a * b = a + b + ab$, a, b in Q.

(a) Both I and II (b) I only

(c) II only (d) None of the above

54. If a is a generator of a cyclic group G of order 8, and if α = order of a^3 and β = order of a^6, then the pair (α, β) =

(a) (4, 2) (b) (4, 8)

(c) (8, 8) (d) (8, 4)

55. Which of the following statements is true ?

I. If p is the smallest prime factor of the order of a finite group G then any subgroup of index p is not always a normal subgroup.

II. A finite group G of even order has no element of order 2.

(a) None (b) I only

(c) II only (d) Both I and II

56. Consider the ring $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$ under the addition and multiplication modulo 8. Which of the following is a unit of the ring X ?

(a) 2 (b) 3

(c) 4 (d) None of the above

57. Which of the following statements is true ?

I. A matrix ring F_n over a field F is not always a simple ring.

II. A division ring is a simple ring.

(a) Both I and II (b) I only

(c) II only (d) None of the above

58. Let V be a vector space of all functions from R to R. If V_1 is the subset of even functions and V_2 the subset of odd functions then :

(a) V_1 and V_2 are not sub - spaces of V

(b) V_1 and V_2 are sub - spaces of V

(c) V_1 is a sub - space of V but V_2 is not a subspace of V

(d) V_2 is a sub space of V but V_1 is not a subspace of V

59. Let $V = \{(a, b) ; a, b \in R\}$. Let the operations of addition and scalar multiplication be given by :

I. $(a, b) + (c, d) = (0, b + d)$ and $k(a, b) = (0, kb)$

II. $(a, b) + (c, d) = (a + c, b + d)$ and $k(a, b) = (|k|a, |k|b)$.

Then V is a vector space under the operations defined by :

(a) I only (b) II only

(c) Both I and II (d) None of the above

60. The set W of all triads defined by $W = \{(x + y, 3y, 2x - y); x, y \in R\}$ is a subspace of $V_3(R)$. What is the dimension of W ?

(a) 2 (b) 3

(c) 1 (d) None of the above

MATHEMATICS

1. For what value of the constant k, the infinitesimals $x^5 - 1$ and $k(x - 1)$, as $x \to 1$, are equivalent ?

   (a) Zero

   (b) 1

   (c) 3

   (d) 5

2. Let $f(x) = e^{-\frac{1}{x}}$ when $x \neq 0$

   $= 0$ when $x = 0$

   and let

   $$g(x) = x \sin \frac{1}{x} \text{ when } x \neq 0$$

   $= 0$ when $x = 0$,

   then which of the following is *true* ?

   (a) $f(x)$ and $g(x)$ are both continuous at $x = 0$

   (b) $f(x)$ is not continuous at $x = 0$, but $g(x)$ is continuous at $x = 0$

   (c) $f(x)$ and $g(x)$ are both discontinuous at $x = 0$

   (d) None of the above

3. What is the coefficient of x^3 in the power series expansion of the function :

   $$\log\left(\frac{1+x}{1-x}\right)^{\frac{1}{3}} \text{ for } -1 < x < 1 \text{ ?}$$

   (a) $\frac{2}{9}$

   (b) $\frac{1}{9}$

   (c) 2

   (d) None of the above

4. Which of the following is the value of the limit

$$\lim_{x \to 0} \frac{\tan x - x + \frac{1}{3}x^3}{x^3} ?$$

(a) 0

(b) $\frac{2}{15}$

(c) $\frac{1}{3}$

(d) $\frac{2}{3}$

5. For which of the following functions :

(I) $f(x) = 1 - (x-1)^{\frac{2}{3}}$ on [0, 2] and

(II) $f(x) = (x+1)^2 (x-2)$ on [–1, 2],

Rolle's theorem is valid ?

(a) For both (I) and (II)

(b) For (I) only

(c) For (II) only

(d) None of the above

6. What is the curvature of the parabola $y^2 = 2px$ at the point $\left(\frac{p}{2}, p\right)$?

(a) $\frac{1}{2\sqrt{2}\,p}$

(b) $\sqrt{2}p^3$

(c) $2p^{\frac{3}{2}}$

(d) None of the above

7. For which of the following values of the pair (k, m), the straight line $kx + m$ is an oblique asymptote to the curve :

$$y = \frac{x^2 + 2x - 1}{x} ?$$

(a) (1, 0)

(b) (1, 2)

(c) (2, 2)

(d) None of the above

8. If $\frac{(1 + i)^9}{(1 - i)^7} = P + iQ$,

then what is the value of P ?

(a) 0

(b) 4

(c) 2

(d) None of the above

9. If $\omega = \cos\frac{2\pi}{3} + i \sin\frac{2\pi}{3}$, then which of the following is the value of $(1 + \omega)^3$?

(a) -1

(b) 1

(c) i

(d) None of the above

10. For what value of the pair (α, β),

$$\cos 40 - 8 \cos^4\theta = \alpha \cos^2\theta + \beta ?$$

(a) (1, 1)

(b) (6, 0)

(c) (8, 0)

(d) (–8, 1)

11. If log (z) denotes the principal value of log(z) and n is an integer, then :

$$\log(-7) - \log 7 =$$

(a) $2n\pi$

(b) $n\pi i$

(c) $(2n + 1)\, i\pi$

(d) None of the above

12. If $C + iS = e^{i\theta}(2 \sin \theta)^{-\frac{1}{2}} (\sin \theta - i \cos \theta)^{-\frac{1}{2}}$, then which of the following is the value of S ?

(a) $\dfrac{\sin\left(\theta + \dfrac{\pi}{2}\right)}{\sqrt{2 \sin \theta}}$

(b) $\dfrac{\sin\left(\dfrac{\theta}{2} + \dfrac{\pi}{4}\right)}{\sqrt{2 \sin \theta}}$

(c) $\dfrac{1}{\sqrt{2}} \cos\left(\dfrac{\theta}{2} + \dfrac{\pi}{4}\right)$

(d) None of the above

13. Which of the following is the equation of the chord of the parabola $y^2 = 8x$ which is bisected at the point $(2, -3)$?

(a) $3x + 4y + 6 = 0$

(b) $4x - 3y - 17 = 0$

(c) $4x - 3y + 31 = 0$

(d) $4x + 3y + 1 = 0$

14. What is the pole of the line $2x + y - 1 = 0$ with respect to the ellipse

$$\frac{x^2}{3} + \frac{y^2}{2} = 1 \text{ ?}$$

(a) (6, 2)

(b) (18, 4)

(c) $(2\sqrt{3}, \sqrt{2})$

(d) None of the above

15. Let P(x, y) be a point on the hyperbola

$$\frac{x^2}{4} - \frac{y^2}{3} = 1.$$

If PS and PS' are focal distances of P(x, y), then $|PS' - PS| =$

(a) 8

(b) 6

(c) 4

(d) None of the above

16. What is the length of the semi-major axis of the ellipse

$$36x^2 + 24xy + 29y^2 - 72x + 126y + 81 = 0$$

given that the centre is the point (2, –3) and the semi-axes of the ellipse are connected by the equation

$$\left(\frac{1}{5} - \frac{1}{r^2}\right)\left(\frac{29}{180} - \frac{1}{r^2}\right) = \frac{1}{225} \text{ ?}$$

(a) 9

(b) 3

(c) 2

(d) None of the above

17. Which of the following is the equation of the plane which meets the coordinate axes in P, Q, R such that the centroid of the ΔPQR is the point (l, m, n) ?

(a) $\frac{x}{l} + \frac{y}{m} + \frac{z}{n} = 3$

(b) $lx + my + nz = 3$

(c) $lx + my + nz = 1$

(d) None of the above

18. Which of the following is the condition that the two spheres :

$$x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$$

and

$$x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$$

be orthogonal ?

(a) $u_1u_2 + v_1v_2 + w_1w_2 = d_1d_2$

(b) $u_1u_2 + v_1v_2 + w_1w_2 = \frac{d_1d_2}{2}$

(c) $u_1u_2 + v_1v_2 + w_1w_2 = \frac{d_1 + d_2}{2}$

(d) None of the above

19. What is the general equation of the cone of second degree which passes through the axes ?

(a) $fx^2 + gy^2 + hz^2 = 0$

(b) $fyz + gzx + hxy = 0$

(c) $ax^2 + by^2 + cz^2 = fyz + gzx + hxy$

(d) None of the above

20. Which of the following is the equation of the plane which cuts the conicoid $x^2 + 4y^2 - 5z^2 = 1$ in a conic whose centre is at the point (2, 3, 4) ?

(a) $x + 4y - 5z + 4 = 0$

(b) $3x + 5y - 4z - 5 = 0$

(c) $x + 4y + 2z - 22 = 0$

(d) $x + 6y - 10z + 20 = 0$

21. What is the value of the integral

$$\int_0^1 \frac{x\,dx}{\sqrt{1-x^4}} ?$$

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{4}$

(c) $\frac{3}{2}$

(d) None of the above

22. For which of the following values of $\phi(x)$,

$$\int x^m \cos nx\,dx + \frac{m(m-1)}{n^2}\int x^{m-2}\cos nx\,dx$$

$$= \frac{x^{m-1}\phi(x)}{n^2} \text{ where } m > 2,\ n \geq 1 ?$$

(a) $(m \sin nx + n \cos nx)$

(b) $n \cos nx - m \sin nx$

(c) $mx \cos nx + n(n-1) \sin nx$

(d) $m \cos nx + nx \sin nx$

23. Given that

$$\int_0^1 \frac{\log (1 + x)}{x} dx = \frac{\pi^2}{12}.$$

What is the value of the integral

$$\int_0^1 \frac{\log x}{1 + x} dx ?$$

(a) $-\frac{\pi^2}{12}$

(b) $\frac{\pi^2}{6}$

(c) $\frac{\pi}{12}$

(d) None of the above

24. For what value of $\phi(x)$,

$$\int_a^b \phi(x)\, dx$$

gives the length of the arc for the curve $y = f(x)$ between two points whose abscissa are a and b where $f'(x)$ is continuous and single valued in the range (a, b) ?

(a) $\left\{1 + (f'(x))^2\right\}^{\frac{3}{2}}$

(b) $\{f(x) + f'(x)\}^{\frac{1}{2}}$

(c) $\left\{1 + (f'(x))^2\right\}^{1/2}$

(d) None of the above

25. The necessary and sufficient condition for a vector function $\vec{f}(t)$ to have constant direction is :

(a) $\vec{f} + \frac{\vec{d}f}{dt} \times \frac{\vec{d^2}f}{dt^2} = 0$

(b) $\vec{f} \times \frac{\vec{df}}{dt} = 0$

(c) $\vec{f} \cdot \frac{\vec{d}f}{dt} = 0$

(d) None of the above

26. If $\vec{f}$ and $\vec{g}$ are continuously differentiable vector point functions, then :

curl $(\vec{f} \times \vec{g}) + (\vec{f} \cdot \nabla)\vec{g} - (\vec{g} \cdot \nabla)\vec{f}$ =

(a) $\vec{f}$ div $\vec{g} - \vec{g}$ div $\vec{f}$

(b) $\vec{f} \times$ curl $\vec{g}$ + curl $\vec{f} \times g$

(c) $\vec{f} \cdot$ curl $\vec{g} - \vec{g} \cdot$ curl $\vec{f}$

(d) None of the above

27. Which of the following is the integrating factor of the differential equation :

$$(1 + x)\frac{dy}{dx} - xy = 1 - x \text{ ?}$$

(a) $e^{-\frac{x^2}{2}}$

(b) e^{-x}

(c) $-x + e^{1+x}$

(d) $e^{-x}(1 + x)$

28. What is the value of

$$\left(\frac{1}{D^2 - 5D + 6}\right) xe^x \ ?$$

(a) $e^x(6x - 5)$

(b) $\frac{e^x}{2}(5x^2 + 6x + 1)$

(c) $\frac{e^x}{4}(2x + 3)$

(d) None of the above

29. Which of the following transformations reduce the differential equation :

$$xp^2 - 2yp + x + 2y = 0, \text{ where } p = \frac{dy}{dx},$$

to Clairaut's form ?

(a) $x^2 = u,\ y^2 = v$

(b) $x^2 = u,\ y - x = v$

(c) $x + y = u,\ xy = v$

(d) None of the above

30. If $P_n(x)$ denote Legendre's polynomials for different values of positive integer n, then for what value of $\psi(n)$:

$$P'_{n+1}(x) - P'_{n-1}(x) = \psi(n)\, P_n(x) \ ?$$

(a) $2n$

(b) $n + 1$

(c) $2n + 1$

(d) None of the above

31. Let

$$G = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -1 \\ 2 & 0 & -4 \end{bmatrix}$$

If H = adjoint of G, what is :

$[H]_{22} + [H]_{32}$ equal to ?

(a) -8

(b) 12

(c) -12

(d) None of the above

32. Let Q be a skew-symmetric matrix of order three, then which of the following is always *true* ?

(a) Q^2 and Q^3 are both skew-symmetric matrices

(b) Q^2 and Q^3 are both symmetric matrices

(c) Q^2 is a skew-symmetric matrix but Q^3 is a symmetric matrix

(d) Q^2 is a symmetric matrix but Q^3 is a skew-symmetric matrix

33. Let

$$G = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix} \text{ and } H = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

then what is the trace GH ?

(a) 30

(b) 11

(c) 13

(d) None of the above

34. For what value of β,

$$\lambda^3 - 12\lambda^2 + \beta\lambda - 32 = 0$$

is the characteristic equation of the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} ?$$

(a) 36

(b) 46

(c) 38

(d) None of the above

35. How many linearly independent solutions does the equation

$$7x + 6y - 11z + 14 = 0$$

have ?

(a) None

(b) One

(c) Two

(d) Three

36. For what value of λ, the column vectors

$$X_1 = \begin{bmatrix} 7 \\ 5 \\ 3 \end{bmatrix}, \; X_2 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, X_3 = \begin{bmatrix} 11 \\ \lambda \\ 0 \end{bmatrix}$$

are linearly dependent ?

(a) 4

(b) 16

(c) –3

(d) None of the above

37. Every polynomial equation of degree $2n + 1$, $n \geq 1$, with real coefficients always has :

(a) no real roots

(b) $2n$ real roots

(c) n real roots

(d) at least one real root

38. Given that $x = -\frac{3}{2}$ is a root of the equation

$$4x^3 + 12x^2 + 11x + 3 = 0,$$

what are its other two roots ?

(a) $-1, 2$

(b) $1, \frac{1}{2}$

(c) $-1, -\frac{1}{2}$

(d) None of the above

39. If $P(z) = z^3 + 3Hz + G$ and $G^2 + 4H^3$ is negative, then the cubic equation $P(z) = 0$ always has :

(a) all the three roots equal

(b) two equal roots

(c) one real and two complex roots

(d) all the three roots real

40. If α, β, γ are the roots of the equation

$$x^3 - 9x^2 + 14x + 24 = 0,$$

what is the value of $1 + \Sigma\frac{1}{\alpha}$?

(a) $\frac{5}{12}$

(b) $\frac{19}{12}$

(c) $\frac{10}{9}$

(d) None of the above

41. Consider the set

$$S = \left\{ \frac{2n+1}{n};\ n \in N \right\}.$$

If l is the supremum and g is the infimum of the set S, then which of the following is *true* ?

(a) l belongs to S whereas g does not

(b) g belongs to S whereas l does not

(c) l and g both belong to S

(d) None of the above

42. The sequence $\{S_n\}$ where

$$S_n = \frac{(-1)^{n-1}}{n!},\ n \in N,$$

is :

(a) not bounded but has one limit point

(b) bounded but has no limit point

(c) bounded and has only one limit point

(d) bounded and has two limit points

43. Let $\sum_1^\infty u_n$ be a positive term series such that :

$$\frac{u_n}{u_{n+1}} = \alpha + \frac{\beta}{n} + \frac{\gamma_n}{n^p},$$

where $\alpha > 0$, $p > 1$ and $\{\gamma_n\}$ is a bounded sequence, then for $\alpha \neq 1$. which of the following is *true* ?

(a) $\sum_1^\infty u_n$ converges if $\alpha < 1$ and diverges if $\alpha > 1$ whatever β may be

(b) $\sum_1^\infty u_n$ converges if $\alpha > 1$ and diverges if $\alpha < 1$ for all real β

(c) $\sum_1^\infty u_n$ converges if $\beta < 1$ and diverges if $\beta > 1$ for all real $\alpha \neq 1$

(d) None of the above

44. Let the function $f(x)$ be defined by

$f(x) = 1$ when x is irrational

$= -1$ when x is rational,

then $f(x)$ is :

(a) continuous at every point

(b) discontinuous at every point

(c) continuous at rationals and discontinuous at irrationals

(d) none of the above

45. If $[x]$ denotes the greatest integer not greater than x, then what is the value of the integral

$$\int_0^3 [x]\, dx \, ?$$

(a) Does not exist

(b) Zero

(c) $\frac{9}{2}$

(d) 3

46. Let $f(x) = \sin x$, $x \in [0, t]$ where $t \leq \frac{\pi}{2}$. Consider a partition

$$p = \left\{0, \frac{t}{n}, \frac{2t}{n}, \frac{3t}{n}, ..., n\frac{t}{n}\right\}$$

of the interval $[0, t]$. Then which of the following is equal to $\frac{n}{t}\{L(p, f)\}$?

(a) $\left(\sin \frac{t}{n}\right) - \frac{n}{t} \cos t$

(b) $\cos \frac{t}{n} + \cos \frac{2t}{n} + ... + \cos \frac{nt}{n}$

(c) $\sin \frac{t}{n} + \sin \frac{2t}{n} + ... + \sin (n - 1) \frac{t}{n}$

(d) $\sin \frac{t}{n} + \sin \frac{2t}{n} + ... + \sin \frac{nt}{n}$

47. Let $f(x, y) = x \sin\left(\frac{1}{y}\right) + y \sin\left(\frac{1}{x}\right), xy \neq 0$

$= 0 \quad , \quad xy = 0$

then what is the value of the limit $\lim_{(x, y) \to (0, 0)} f(xy)$?

(a) Does not exist

(b) ∞

(c) 2

(d) Zero

48. Let $f(x, y) = \frac{x^2y^2}{x^2 + y^2}, (x, y) \neq (0, 0)$

$= 0, \quad x = y = 0$

If $l = f_x(0, 0)$ and $m = f_x(2, 1)$, what is the value of the pair (l, m) ?

(a) $\left(0, \frac{4}{25}\right)$

(b) (0, 1)

(c) $\left(0, \frac{4}{5}\right)$

(d) None of the above

49. If $x = r \cos \theta$ and $y = r \sin \theta$, which of the following is equal to the pair $\left(\frac{\partial r}{\partial x}, \frac{\partial \theta}{\partial y}\right)$?

(a) $\left(\sec \theta, \frac{\sec \theta}{r}\right)$

(b) $\left(\sec \theta, \frac{\cos \theta}{r^2}\right)$

(c) $\left(\cos \theta, \frac{\cos \theta}{r}\right)$

(d) None of the above

50. If

$$F(x, y) = x^2 + y^2 + \lambda (x^2 + 8xy + 7y^2 - 225),$$

then what is the value of d^2F at $\lambda = -\frac{1}{9}$ where $\frac{\partial F}{\partial x} = 0$ and $\frac{\partial F}{\partial y} = 0$?

(a) $\frac{4}{9}\left(4(dx)^2 + (dy)^2\right)$

(b) $\frac{4}{9}(2dx - dy)^2$

(c) $\frac{16}{9}(dx)^2$

(d) None of the above

51. What is the value of the double integral

$$\iint_R (x^2 + y^2)dxdy$$

where $R = \left\{(x, y);\ x^2 + y^2 \leq \sqrt{2}\right\}$?

(a) π

(b) 2π

(c) $2\sqrt{2}\pi$

(d) None of the above

52. Which of the following is the value of the triple integral

$$\iiint_R xyz\, dxdydz$$

where

$$R = \{(x, y, z); 0 < z < y, 0 < y < x, 0 < x < 1\} ?$$

(a) $\frac{1}{8}$

(b) $\frac{1}{24}$

(c) $\frac{1}{48}$

(d) None of the above

53. Let Z_1 be the set of all non-negative integers and Q_1 be the set of all non-zero rational numbers, then which of the following is *true* ?

(a) Z_1 is not a group under addition composition but Q_1 is a group under multiplication composition

(b) Q_1 is not a group under multiplication composition but Z_1 is a group under addition composition

(c) Z_1 and Q_1 are groups under the composition of addition and multiplication respectively

(d) None of the above

54. Let Z be the additive group of integers and let H_t denote the set of all multiples of a positive integer t then which of the following is *true* ?

(a) H_{15} is not a subgroup of Z but H_{19} is a subgroup of Z

(b) H_{15} is a subgroup of Z but H_{19} is not a subgroup of Z

(c) Both H_{15} and H_{19} are not subgroups of Z

(d) Both H_{15} and H_{19} are subgroups of Z

55. Let Z be the set of all integers and G = {1, −1}. Here G is a group under multiplication. Define a mapping

$$f : Z \to G$$

by putting $f(n) = 1$ if n is even and $f(n) = -1$ if n is odd. Then which of the following is *true* ?

(a) $f(m + n) \neq f(m)\ f(n)$ for all m, n in Z

(b) $f(m + n) = f(m)\ f(n)$ for all m, n in Z but f is not onto

(c) $f(m + n) = f(m)\ f(n)$ for all m, n in Z and f is also onto

(d) None of the above

56. What is the number of distinct cycles of length 4 in S_9, the symmetric group of degree 9 ?

(a) 3024

(b) 126

(c) 36

(d) 756

57. A ring with unity is called a division ring if :

(a) it has proper zero divisors

(b) its all non-zero elements form a group under multiplication

(c) its all non-zero elements form a group under addition

(d) None of the above

58. Let

$$R_1 = \left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \middle| a \in R \right\},$$

where R is a ring. Define a mapping $f : R_1 \to R$ by

$$f\left(\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}\right) = a \text{ for all } \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \in R_1,$$

then which of the following is *true* ?

(a) $f(X + Y) \neq f(X) + f(Y)$ for all X, Y in R_1

(b) $f(XY) \neq f(X)f(Y)$ for all X, Y in R_1

(c) $f(XY) \neq f(X)f(Y)$ but $f(X + Y) = f(X) + f(Y)$ for all X, Y in R_1

(d) $f(XY) = f(X)f(Y)$ and $f(X + Y) = f(X) + f(Y)$ for all X, Y in R_1

59. Let $P = R[x]$ and let $f(x)$, $g(x)$ $h(x)$, $k(x) \in P$ be such that

$$f(t) = 1,\ g(t) = t,\ h(t) = t^2,\ k(t) = 1 + t + t^2$$

for all $t \in R$, then which of the following is *true* ?

(a) $f(x)$, $g(x)$, $h(x)$, $k(x)$ are linearly independent

(b) $f(x)$, $g(x)$,$h(x)$ are linearly independent

(c) $f(x)$, $g(x)$, $h(x)$ are linearly dependent

(d) None of the above

60. Let C be the field of complex numbers. We can regard C as a vector space over itself. Define $T : C \to C$ such that :

$$T(Z) = x \text{ for any } Z = x + iy,\ x, y \in R,$$

then which of the following is *true* ?

(a) $T((2 + i)(2 - i)) \neq (2 + i)\ T\ (2 - i)$

(b) $T((2 + i)(2 - i)) = (2 + i)\ T(2 - i)$

(c) $T(Z_1 + Z_2) \neq T(Z_1) + T(Z_2)$, where $Z_1 = x_1 + iy_1$, $Z_2 = x_2 + iy_2$, $x_1, x_2, y_1, y_2 \in R$

(d) None of the above

HOME SCIENCE

1. Polyandry refers to :

   (A) one man marrying one woman

   (B) one man marrying more than one woman

   (C) one woman marrying more than one man

   (D) none of the above

2. A value of variable which divides the series in such a manner that number of items below it is equal to the number of items above it, is called :

   (A) Mean

   (B) Mode

   (C) Median

   (D) Average

3. Which of the following is *not* a measure of dispersion ?

   (A) Range

   (B) Mean Deviation

   (C) Standard Deviation

   (D) Class Interval

4. The UN resolution on discrimination against women was adopted in :

   (A) 1967

   (B) 1963

   (C) 1975

   (D) 1981

5. Who was the Chairman of Indian Education Commission ?

   (A) D.S. Kothari

   (B) J.P. Naik

   (C) Hamid Ansari

   (D) None of the above

6. Integration of pupils with learning difficulties into regular class rooms for all or a part of school day is called :

(A) Guidance

(B) Mainstreaming

(C) Counselling

(D) Directing

7. Down's syndrome refers to :

(A) Nutritional Deficiency

(B) Infection

(C) Temp. Shock

(D) Chromosomal Abnormality

8. Which of the following are known as protective foods ?

(A) Fruits

(B) Fats and oils

(C) Cereals

(D) All of the above

9. Which of the following should necessarily be supplied through diet ?

(A) Short chain fatty acids

(B) Long chain fatty acids

(C) Essential fatty acids

(D) Non-essential fatty acids

10. Which of the following is a water soluble vitamin ?

(A) Vit. E

(B) Vit. D

(C) Vit. A

(D) Vit. C

11. Major part of wheat flour is obtained from which part of the seed ?

(A) Endosperm

(B) Germ

(C) Aleuron layer

(D) Scutellum

12. Which of the following are filamentous ?

(A) Bacteria

(B) Yeast

(C) Fungi

(D) None of the above

13. Who is the basic functionary in a community development programme ?

(A) Gram Sewak

(B) BDO

(C) District Collector

(D) None of the above

14. In Panchayati Raj, the Sarpanch is elected by :

(A) People

(B) Panchayat Members

(C) MLAs

(D) Nominated by DC

15. In a communication model, which of the following is the first element of communication ?

(A) Treatment

(B) Message

(C) Sender

(D) Channel

16. Symposia are the examples of :

(A) Mass approach

(B) Group approach

(C) Individual approach

(D) None of the above

17. Which of the following statements about a poster is false ?

(A) Always write captions vertically

(B) Bring out the message clearly

(C) Use bright attractive colours

(D) It should recommend action

18. Film projectors are included in :

(A) Reflected projectors

(B) Indirect projectors

(C) Direct projectors

(D) None of the above

19. Which of the following have a background of coloured cloth ?

(A) White board

(B) Black board

(C) Bulletin board

(D) All of the above

20. Which of the following is *not* a poverty alleviation programme ?

(A) NREP

(B) TRYSEM

(C) IRDP

(D) ICDS

21. Which of the following is *not* a component of staffing ?

(A) Placement

(B) Transfers

(C) Directing

(D) Training

22. Write the odd one :

(A) Knowledge

(B) Money

(C) Skills

(D) Interest

23. The measurement of the amount of light emitted by a bulb is called its :

(A) Lumen

(B) Watt

(C) Wattage

(D) Reflection

24. Which of the following soil types can bear maximum pressure of building on it ?

(A) Hard clay

(B) Sandy clay

(C) Sandy loams

(D) Moist soils

25. Which of the following statements pertain to organismic theory/theories of child development ?

(A) Psychological structures exist inside

(B) Change is stimulated by environment

(C) Both the above

(D) None of the above

26. Directly observable features of a person are known as :

(A) Prototype

(B) Phenotype

(C) Genotype

(D) Karyotype

27. Each cell in our body contains :

(A) 23 pairs of genes

(B) 46 pairs of genes

(C) 23 pairs of chromosomes

(D) 46 pairs of chromosomes

28. Which of the following is an X linked disease ?

(A) Hemophilia

(B) Cystic fibrosis

(C) PKU

(D) Sickle Cell Anemia

29. Palmer grasp disappears at the age of :

(A) 2 years

(B) 18 months

(C) 1 year

(D) 3-4 months

30. An organized pattern of physical growth and motor control that proceeds head to tail is called :

(A) Cephalocaudal trend

(B) Cephalothoracic trend

(C) Proximodistal trend

(D) Dynamic systems theory

31. A standard of fairness in which individuals express the same concern for the welfare of others as they do for themselves is known as :

(A) Reciprocity

(B) Autonomous morality

(C) Realism

(D) Heteronymous morality

32. Vishwa Bharati at Shantiniketan was established by :

(A) Gandhiji

(B) Rabindranath Tagore

(C) G.K. Gokhale

(D) Jawahar Lal Nehru

33. The feeling of satisfaction which a person derives out of the use of a good or service is called :

(A) Psychic income

(B) Real income

(C) Money income

(D) None of the above

34. Which of the following acts protects consumers from hazards of food adulteration ?

(A) Dangerous Drug Act

(B) Weights and Measurement Act

(C) FPO

(D) PFA

35. In which of the following the return is in the form of profit ?

(A) Bonds

(B) Shares

(C) Both the above

(D) None of the above

36. Windows placed on the walls raised above the roof to allow light and air is called :

(A) Clerestory

(B) Ventilator

(C) Exhaust

(D) Aerator

37. Which of the following is an alkaline salt of fatty acid ?

(A) Washing soda

(B) Detergents

(C) Disinfectants

(D) Soaps

38. Which of the following is used as an egg beater ?

(A) Whisks

(B) Mixers

(C) Grinders

(D) All of the above

39. Which of the following is *not* a component of a refrigerator ?

(A) Beater

(B) Compressor

(C) Expansion valve

(D) Gasket

40. The ironing surface of an electric iron is known as :

(A) Sole plate

(B) Hot plate

(C) Heating mantle

(D) Thermoregulator

41. Which of the following is the more appropriate appliance to heat a Samosa or a patti ?

(A) Hot air oven

(B) Microwave oven

(C) BOD incubator

(D) Toaster

42. Which of the following is *not* a primary colour ?

(A) Red

(B) Yellow

(C) Blue

(D) Green

43. Staple is associated with :

(A) Dimensions of fibre

(B) Elasticity of fibre

(C) Colour of fibre

(D) All of the above

44. Cotton is an example of ..... fibre.

(A) Protein

(B) Mineral

(C) Cellulosic

(D) Thermoplastic

45. The process of pressing the fabric to smooth out wrinkles and add sheen to it is known as :

(A) Beetling

(B) Embossing

(C) Sanforising

(D) Calendering

46. Which of the following is *not* used for solubility test of fibres ?

(A) Ethanol

(B) Acetone

(C) Phenol

(D) Conc. Sulphuric acid

47. Which of the following absorbs moisture readily ?

(A) Linen

(B) Cotton

(C) Nylon

(D) Non-difference

48. Which of the following laundry blues are insoluble in water ?

(A) Ultramarine

(B) Prussian blue

(C) Methylene blue

(D) Coal tar dyes

49. Energy required by the body when lying at rest in a comfortable environment, mentally relaxed and without food is known as :

(A) RDI

(B) SDA

(C) RDA

(D) BMR

50. Which of the following yields highest energy in the body ?

(A) Vitamins

(B) Carbohydrates

(C) Protein

(D) Fat

51. Calcium requirements increase in :

(A) Third trimester of pregnancy

(B) Second trimester of pregnancy

(C) First trimester of pregnancy

(D) Do not change

52. Protein requirement during infancy (first 6 months) is :

(A) 10 g/kg body weight

(B) 15 g/kg body weight

(C) 5 g/kg body weight

(D) 2 g/kg body weight

53. Which of the following is a monosaccharide ?

(A) Glucose

(B) Lactose

(C) Sucrose

(D) Maltose

54. Which of the following is starch ?

(A) Lactose

(B) Cellulose

(C) Pectin

(D) Amylopectin

55. Which of the following is a saturated fatty acid ?

(A) Palmitic acid

(B) Oleic acid

(C) Linoleic acid

(D) Linolenic acid

56. Which of the following is a sulphur containing amino acid ?

(A) Cystein

(B) Cystine

(C) Methionine

(D) All of the above

57. Enzymes are ..... in nature.

(A) Proteins

(B) Carbohydrates

(C) Fats

(D) None of the above

58. Which of the following need metabolic energy ?

(A) Diffusion

(B) Passive transport

(C) Active transport

(D) All of the above

59. Which of the following catalyse blood clotting ?

(A) Calcium

(B) Fluoride

(C) Copper

(D) All of the above

60. National Institute of Nutrition (NIN) is located at :

(A) Hyderabad

(B) Bangalore

(C) Mysore

(D) Delhi

HOME SCIENCE

1. Which of the following statements is not true about ovum :

   (A) Contains 23 chromosomes

   (B) Develops in ovaries

   (C) Elongated with a fine hair like tail

   (D) Contains yolk

2. In human development, the germinal stage lasts for ..... after fertilization.

   (A) 10 days—10 weeks

   (B) 10 days—2 weeks

   (C) 4 weeks—6 weeks

   (D) 12 weeks—20 weeks

3. An average baby can walk with help at the age of :

   (A) 6—8 months

   (B) 4—6 months

   (C) 16—18 months

   (D) 9—11 months

4. A child starts making use of holophrases at the age of :

   (A) 1 year

   (B) 2 years

   (C) 6 months

   (D) 4 months

5. The psychosocial theory of personality development was given by :

(A) Erik Erikson

(B) Sigmund Freud

(C) Yarrow

(D) Pederson

6. A child is in anal stage from :

(A) 1½-3 years

(B) Birth-1½ years

(C) 3-4 years

(D) 4-5 years

7. Slanting eyes is a feature of :

(A) Cretinism

(B) Microcephaly

(C) Mongolism

(D) Albinism

8. A child plays independently among other children and does not influence or modify the activity of other children. The play is :

(A) Solitary independent play

(B) Associative play

(C) Parallel play

(D) Organized supplementary play

9. When parents try to control their children's behaviour and make them conform to a set and usually absolute standard of conduct, they are :

(A) Authoritarian parents

(B) Authoritative parents

(C) Permissive parents

(D) None of the above

10. Enuresis refers to :

(A) Sleeplessness

(B) Bed wetting;

(C) Down's syndrome

(D) Night blindness

11. Resilience refers to ..... of fiber.

(A) Flexibility

(B) Strength

(C) Water absorption

(D) Springiness

12. Which of the following methods is used for determining hair weight of textiles ?

(A) Clegg's method

(B) Winson's method

(C) A.S.T.M. method

(D) All of the above

13. Fells are the fabrics made by :

(A) Passing one set of yarns above and below another

(B) Twisting many fibers about each other

(C) Looping one yarn about itself

(D) Uniting loose fibers

14. The number of units of weight in a unit of length is given by :

(A) Denier

(B) Plies

(C) Crimp

(D) None of the above

15. Which of the following is a cellulosic material ?

(A) Cotton

(B) Silk

(C) Wool

(D) None of the above

16. Which of the following needs weight and height for its calculation ?

(A) BMR

(B) BMI

(C) PER

(D) BV

17. Which of the following is *not* used for anthropometric methods of nutritional assessment ?

(A) Height

(B) Mid arm circumference

(C) Head circumference

(D) Haemoglobin

18. The major objective of mid day meal programme is :

(A) To provide minerals to infants

(B) To provide vit. D to pregnant ladies

(C) To provide protein to poor children

(D) To reduce school dropouts

19. Potable water refers to :

(A) Water that can be transported

(B) Water which contains minerals

(C) Water which is safe for drinking

(D) Water which contains pathogens

20. Which of the following pertains to food adulteration ?

(A) PFA

(B) FPO

(C) MFPO

(D) All of the above

21. Which of the following is a water soluble vitamin ?

(A) Vit. A

(B) Vit. C

(C) Vit. D

(D) All of the above

22. Which of the following components of wheat grain contributes to flour ?

(A) Endosperm

(B) Germ

(C) Epicotyle

(D) Hypocotyle

23. Which of the following is a pome fruit ?

(A) Apple

(B) Grapes

(C) Banana

(D) Strawberry

24. Which of the following is comprised of glucose and galactose ?

(A) Sucrose

(B) Lactose

(C) Maltose

(D) None of the above

25. Which of the following undergoes saponification ?

(A) Fatty acids

(B) Glycogen

(C) Cellulose

(D) Pectin

26. Which of the following is the activity of management ?

(A) Planning

(B) Organizing

(C) Leading

(D) All of the above

27. The lumen is the measurement of :

(A) Light emitted

(B) Power consumed

(C) Heat emitted

(D) All of the above

28. A continuous magnet is embedded in the gasket of the refrigerator :

(A) To attract metallic pieces from food

(B) To prevent chilling injury

(C) To hold the door of refrigerator tightly

(D) To reduce the temperature

29. The sole plate of electric iron is coated with :

(A) Chromium

(B) Silver

(C) Iron

(D) Foam

30. An education which is institutional activity, subject oriented, full time and leads to certificates and degrees is called :

(A) Technical education

(B) Extension education

(C) Non-formal education

(D) Formal education

31. Rural reconstruction institute was started by Rabindra Nath Tagore at :

(A) Lucknow

(B) Gurgaon

(C) Shantiniketan

(D) Delhi

32. Which of the following is *not* a characteristic of extension ?

(A) Cooperative

(B) Informal

(C) Nonflexible

(D) Voluntary

33. Any change of behaviour which takes place as a result of experience may be called :

(A) Learning

(B) Formal education

(C) Adult education

(D) None of the above

34. Polyandry refers to :

(A) A man marrying more than one wife

(B) A woman marrying more than one husband

(C) Having many children

(D) All of the above

35. Which of the following defined communication as "Process by which two or more people exchange ideas, facts, feelings or impressions in" a way that each gains common understanding of message" ?

(A) Leagans

(B) Coleman

(C) Schramm

(D) Loomis

36. Tick the odd one out :

(A) Frequency curves

(B) Frequency polygons

(C) Continuous curves

(D) Pictograms

37. Free radicle theory pertains to :

(A) Aging

(B) Child development

(C) Infancy

(D) Adolescence

38. Insomnia refers to :

(A) Lack of appetite

(B) Lack of thirst

(C) Inability to sleep

(D) Inability to breathe

39. The greek word 'geron' means :

(A) Old man

(B) Young man

(C) Young woman

(D) Infant

40. Zygote refers to :

(A) Male sex cell

(B) Female sex cell

(C) Cell formed as a result of fertilization

(D) None of the above

41. Neonate is the baby in :

(A) First month of life

(B) First year of life

(C) First two years of life

(D) First three years of life

42. The first conference on women was held in :

(A) 1975

(B) 1965

(C) 1955

(D) 1985

43. In India, the community development programme was launched in :

(A) 1975

(B) 1965

(C) 1947

(D) 1952

44. Krishi Vigyan Kendras are established by :

(A) ICAR

(B) ICMR

(C) ICCR

(D) NIN

45. Which of the following is an audio-visual aid ?

(A) Poster

(B) Radio

(C) Television

(D) Black-board

46. Which of the following is *not* a poverty alleviation programme ?

(A) IRDP

(B) JRY

(C) NREP

(D) ICDS

47. Which of the following is an unsaturated fatty acid ?

(A) Caproic acid

(B) Caprylic acid

(C) Capric acid

(D) Oleic acid

48. Which of the following contains a pyrole ring ?

(A) Carotene

(B) Vit. C

(C) Sucrose

(D) Hemoglobin

49. The sequence of amino acids linked by peptide bonds refers to which of the following structures of protein ?

(A) Primary

(B) Secondary

(C) Tertiary

(D) Quaternary

50. The non-protein organic component of enzymes is called :

(A) Zymogen

(B) Prosthetic group

(C) Apoenzyme

(D) Isozyme

51. Fat is stored in :

(A) Adipose tissue

(B) Neural tissue

(C) Muscle tissue

(D) Epithelial tissue

52. Hemoglobin contains :

(A) Iron

(B) Magnesium

(C) Calcium

(D) All of the above

53. Glomerulus is present in :

(A) Axon

(B) Neuron

(C) Dendron

(D) Nephron

54. Fundus is a part of :

(A) Kidney

(B) Stomach

(C) Lung

(D) Brain

55. Which of the following is associated with dental caries :

(A) Iodide

(B) Chloride

(C) Fluoride

(D) Bromide

56. National Institute of Nutrition is affiliated to :

(A) ICMR

(B) UGC

(C) ICAR

(D) None of the above

57. Demonstrating the jam making rural women is an example of :

(A) Method demonstration

(B) Result demonstration

(C) Technical demonstration

(D) All of the above

58. Which of the following is Vit. C ?

(A) Calciferol

(B) Tocopherol

(C) Ascorbic acid

(D) β-carotene

59. Which of the following is most suitable for delivering a lecture ?

(A) MS word

(B) MS Axel

(C) MS Power Point

(D) MS Access

60. Which of the following is used in jam making ?

(A) Gelatin

(B) Starch

(C) Pectin

(D) All of the above